GENERALIZED ENTROPY AS A POSSIBLE METRIC FOR MECHANISM EFFICIENCY

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ABSTRACT

Entropy today is a standard uncertainty measure, whose roots date back to Boltzman. Within the information theory context, Shannon in the late 40s, placed entropy on firmer grounds. Shannon's entropy helped us grasp the world around us in a new perspective, which lasted until 1988. Inception of Tsallis or generalized entropy showed that Shannon's extensive entopy is valid through a very narrow window. Interactions among subsystems either dissipates or generates entropy. This is the motivation behind this paper that this new metric presents a potential in engineering systems with intertwined components, whose efficiency is a function of the subsystem interactions.

Keywords: Shannon's Entropy, Tsallis Entropy, Nonextensivity

1. INTRODUCTION

Entropy has been in use for decades in a wide range of areas. It has its origins back in the 19th century, dating back to Boltzmann. His formulation of entropy was meant to quantify the disorder that every physical system strives to maximize. Considered as the cornerstone in statistical mechanics, it has been put to use from black holes to chemistry. In time, various new formats have also been introduced. Shannon, and Renyi are just to name a few. Shannon had proposed [1]:

$$H(X) = -\sum_{r} p(r)\log_2 p(r) \tag{1}$$

p(r) is the probability of a certain signal to appear. Please note that the eventual entropy is positive in sign.

Another one is gaining recognition and known as Tsallis entropy [2]. Tsallis is different from the conventional entropy definition in that it is nonextensive, meaning the entropy of an entire system no longer equals the sum of the entropies of its various parts. This property is displayed by the systems on the verge of chaos. Tsallis entropy has already been applied to research from the locomotion of microorganisms to the collisions of subatomic particles.

2. TSALLIS ENTROPY

Tsallis entropy elaborates extensive and non extensive systems. So before further ado, extensivity and non extensivity must be clearly defined. Boltzmann-Gibbs statistical mechanics and standard thermodynamics are not universal approaches to explain all the systems. So there is need to have a different approach to non extensive systems. It has been clarified that the microscopic dynamics affects the statistical behavior of a physical system. Consequently, it is important to understand the boundaries of the traditional Boltzmann-Gibbs statistical mechanics.

Entropy is supposed to be an extensive property that its value depends on the quantity of the material in the system. Constantino Tsallis came up with non extensive entropy, which is a generalization of the well known Boltzmann-Gibbs entropy. The main idea behind the theory is that Boltzmann-Gibbs entropy assumes that the systems have a strong dependence on initial conditions. In reality most systems behave quite independently from initial conditions. Non extensive entropy accepts non extensive statistical mechanics, whose typical functions are power laws, instead of the traditional exponentials.

In the literature, Tsallis entropy has been applied in wide ranging areas. For example, S.Tong, A. Bezerianos, R.Geocadin, D.Hanley and N.Thakor [3] calculated the Tsallis entropy for brain signals (EEG). O.A. Rossoa, M.T. Martinb, A. Plastino [4] used non extensive information measurement to reach order and maximal complexity of the system with using different complexity types. M. Portes de Albuquerque, I.A. Esquef, A.R. Gesualdi Mello and M. Portes de Albuquerque in their work [5] defined Tsallis entropy as an image segmentation technique. Peng Zhao, Peter Van-Eetvelt, Cindy Goh, Nigel Hudson, Sunil Wimalaratna, and Emmanuel Ifeachor suggested the Tsallis's approach for Alzheimer diseases [6]. Papadimitriou, M. Kalimeri, and K. Eftaxias used the Tsallis entropy for electromagnetic emissions [7].

Interestingly, the generalized entropy has yet failed to find a niche in a mechanical engineering application, where almost no application exists. This work intends to discuss the potentials with the generalized entropy in multi-component mechanical systems.

4. THE IMPORTANCE OF TSALLIS ENTROPY

A nonextensive framework has been proposed by Tsallis 1988. Nonextesivity emerges with memory effects and long-range correlations. Tsallis entropy is also called the generalized entropy, encompassing Shannon entropy [2]:

$$TE(A) = \frac{1 - \sum_{i}^{M} p_i^q}{q - 1} \tag{2}$$

When the nonextensivity parameter q is extracted for the case where there are two identical subsystems (A and B) then :

$$q = 1 + \frac{2TE(A) - TE(A \cup B)}{TE^2(A)} \tag{3}$$

4.1. Superextensivity, q<1

A system that increases subsystem entropies are regarded in this case. A certain type of "mismatch" must occur between these two submechanisms, so that when combined, more information is lost as a whole.

4.2. Extensivity, q=1

information is not lost or gained under unification. This case corresponds to Shannon entropic form of extensivity. The real world is rather far from being extensive.

4.3. Subextensivity, q>1

Consider a case where information is gained upon the unification of the subsystems. In other words, more is known about these two subsystems when combined.

5. AN EXAMPLE AND A QUESTION

This is a realm unfortunately even the author of these lines does not know much about. Still being discussed in the circles of physicists, the ramifications have not yet fully descended on the engineering domain. We know today that the isolated view of the ideal gas is a picture of a locally interacting gas, which is far from the actual gas, having local and/or long-range interactions. Long-range interactions generate effects that we have only started to understand recently. Brain dynamics change as the different regions have interactions [8]. The onset of epilepsy reduces the degree of freedom of the human brain, so to speak, reducing also the complexity of the human brain. Similarly, all links in a mechanism are considered to be non-interactive, other than the forces and torques at the points of contact, i.e. joints. What if non-touching links have a way of affecting each other, such as through vibrations, or magnetism, so that the overall performance of the mechanism may be enhanced or deteriorated. Observations, and the work in physics corroborate the proposition in this contribution. Equation 3 could easily attest to that. Next work is on generating an expression of entropic uncertainty based on geometry, state space, and manufacturing issues such as backlash, etc. to verify this notion on mechanisms.

CONCLUSIONS

For a given number of states of a subsystem A, one gets a higher entropy if q decreases. Or conversely, entropy decreases as q increases. As the number of interactions increase indefinitely, entropy approaches zero. This fact reveals that interactions may be sources of information. More information is gained (or lost) as more interactions take place. This reduction of entropy could also be explained through the free parameters (DoF). Free parameters diminish in number if two or more subsystems get more entangled, or coupled, which, in turn, translates that knowing all about one element would correspond to knowing all about the rest.

This concept is important when a mechanical system is composed of multiple stages or subsystems. Tsallis generalized entropy suggests that knowing the efficiency of each subsystem to compute the overall efficiency of the system is not enough. One must also know, especially long range, subsystem to subsystem interactions so that one can say with certainty that if additional uncertainty is gained or not. Even though the author does not possess any sound data, the author suspects that these interactions may be observed in applications from gearboxes to structural elements connected side by side, where a physical, and whatnot, couplings is under question among many subsystems.

In this work, a connection between statistical mechanics and mechanical systems is tried to be established. Even though these two realms seem to be worlds apart, there is a good amount of potential that all the intertwined inner "mechanical information" transfer defines the eventual quality of the performance of a certain system, mechanism or a machine. This notion does makes sense, but it must surely be put to test on a real setup.

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