TRANSIENT FREE CONVECTION OF AL_2O_3 NANOFLUID IN A SQUARE CAVITY

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ABSTRACT

In this work, the transient free convection of Water – Al_2O_3 nanofluid in two dimensional square cavity is studied numerically using a finite element method. The top and the bottom walls are considered insulated; the two vertical walls are maintained at constant cold and hot temperature respectively. The unsteady state form of pressure-velocity for the Navier–Stokes equations and energy equation are used to model the momentum and energy conservations of the nanofluid medium in the enclosure. The finite element formulations of the dimensionless governing equations with the associated boundary conditions are solved by using the Characteristic-Based Split (CBS) algorithm with linear triangular element discretization scheme for all the field variables. The Rayleigh number is varied from 10^2 to 10^5 whereas the volume fraction of (nanoparticles) solid/ water is varied from 0 to 10%. The results are presented in the form of velocities and isotherms plots as well as the variation of the average Nusselt number with time. It is found that the average Nusslet number decreases as the solid volume fraction increase for any value of Rayleigh number.

Keywords: Nanofluid, Free convection, Transient, Al_2O_3, FEM.

1. INTRODUCTION

One of the most important phenomena in thermal engineering systems is the natural convection heat transfer; this is due to its wide applications in electronics cooling, heat exchangers, and phase change materials [1–3]. The conventional heat transfer fluids have low thermal conductivity that can be improved by using a dispersion of nanonparticles in the base fluid. Such fluids which known as nanofluids, have been studied extensively during the recent years [4–7]. The natural convection heat transfer research has received less attention than the forced convection. Therefore, it is still required to study the effect of the nanoparticles on the heat transfer enhancement in natural convection applications.

However, the results conflict about how the nanoparticle affects the heat transfer, some workers reported that the existence of nanoparticle increase the heat transfer [8,9] whereas other workers reported that the nanofluids have heat coefficient lower than that for a clear fluid [10].

A review on the recent researches on theoretical and numerical investigations discussed and summarized the effect of various thermal properties and applications of nanofluids on the heat transfer analysis [11].

A little effort has been spent to study the transient behavior of the natural convection in nanofluids. In this work an attempt is made to study the effects of Rayleigh number and the solid volume fraction on temperature, velocities, pressure distribution and the Nusslet number in a differentially heated square cavity under unsteady state conditions.

2. MODEL FORMULATION

A schematic of the two-dimensional system with geometrical and boundary conditions is shown in Fig.(1). In the present analysis, Cartesian coordinate system will be applied to the enclosure. The nanofluid in the enclosure is Newtonian, incompressible, and laminar and is assumed to have uniform shape and size. The nanofluid used, which is composed of Aluminum oxide nanoparticles in suspension of Water, has been used at various particle concentrations ranging from 0 to 10% in volume. Moreover, it is assumed that both fluid phase and nanoparticles are in thermal equilibrium state. The left wall is heated and maintained at a constant temperature (T_h) higher than the right cold wall temperature (T_c) which equals to the environment temperature ($T_c = T_\infty$). The thermo-physical properties of the nanofluid are listed in Table (1),[4].

Table (1) :Thermophysical properties of Aluminum oxide and water.

Property	Al_2O_3	Water
Cp (J/kg K)	765	4179
ρ (kg/m^3)	3960	997
k (W/m K)	25	0.61
β (1/K)	0.8×10^{-5}	2.1×10^{-4}
μ (kg/m s)		0.85×10^{-3}

The viscosity and the thermal conductivity of the nanofluid are given by the following models,[11]:

$$\frac{k_n}{k_f} = \frac{k_p + 2k_f - 2(k_f - k_p)\varphi}{k_p + 2k_f + (k_f - k_p)\varphi} \tag{1}$$

$$\mu_n = \frac{\mu_f}{(1-\varphi)^{2.5}} \tag{2}$$

The governing equations for a Boussinesq incompressible laminar flow under unsteady state conditions in a two-dimensional geometry take the following form:

Continuity equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3}$$

Momentum equations:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho_n}\frac{\partial p}{\partial x} + \frac{\mu_n}{\rho_n}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{4}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho_n}\frac{\partial p}{\partial y} + \frac{\mu_n}{\rho_n}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + g\beta_n(T - T_\infty) \tag{5}$$

Energy equation:

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_n\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{6}$$

The following dimensionless variables are used:

$$X = \frac{x}{L}; \quad Y = \frac{y}{L}; \quad \tau = \frac{t\alpha_f}{L^2}$$

$$U = \frac{uL}{\alpha_f}; \quad V = \frac{vL}{\alpha_f}; \quad P = \frac{pL^2}{\rho_f {\alpha_f}^2}; \quad \theta = \frac{T - T_c}{T_h - T_c}$$

$$k = \frac{k_n}{k_f}; \quad \alpha = \frac{\alpha_n}{\alpha_f}; \quad \mu = \frac{\mu_n}{\mu_f};$$

The governing equations are re-written in above dimensionless form as follows:

$$\frac{\partial U}{\partial \tau} + U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = -\frac{1}{\Phi(\rho)}\frac{\partial P}{\partial X} + Pr\frac{\mu}{\Phi(\rho)}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \tag{7}$$

$$\frac{\partial V}{\partial \tau} + U\frac{\partial V}{\partial x} + V\frac{\partial V}{\partial y} = -\frac{1}{\Phi(\rho)}\frac{\partial P}{\partial Y} + Pr\frac{\mu}{\Phi(\rho)}\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + RaPr\Phi(\beta)\theta \tag{8}$$

$$\frac{\partial \theta}{\partial \tau} + U\frac{\partial \theta}{\partial x} + V\frac{\partial \theta}{\partial y} = \frac{k}{\Phi(\rho C_p)}\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) \tag{9}$$

where

$$\Phi(\lambda) = 1 - \varphi + \varphi\frac{\lambda_p}{\lambda_f}$$

is the property function; e.g;

$$\Phi(\beta) = 1 - \varphi + \varphi\frac{\beta_p}{\beta_f}$$

The following dimensionless initial boundary conditions are imposed:

Initial boundary

$$at\ \tau = 0: \theta = 0;\ U = 0;\ \ V = 0 \tag{10}$$

Boundary conditions

$$at\ X = 0: \theta = 1;\ U = 0;\ \ V = 0 \qquad left\ side$$

$$at\ X = 1: \theta = 0;\ U = 0;\ \ V = 0 \qquad right\ side \tag{11}$$

$$at\ Y = 0\ and\ Y = 1;\ \frac{\partial \theta}{\partial y} = 0;\ U = 0;\ \ V = 0 \qquad upper\ and\ lower\ sides$$

The local Nusselt number along the hot wall is defined as

$$Nu = \frac{-L}{(T_h - T_c)} \frac{\partial T}{\partial x}\bigg|_{x=0} = -\frac{\partial \theta}{\partial X}\bigg|_{X=0} \tag{12}$$

and the average Nusselt number is expressed as

$$Nu_{av} = \int_0^1 Nu \, dY. \tag{13}$$

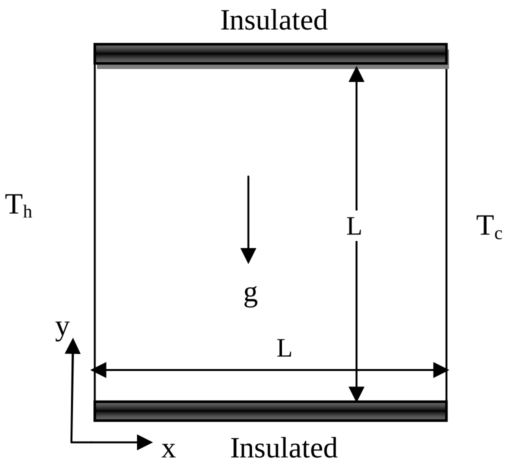

Fig.(1) : Physical model.

3. NUMERICAL SOLUTION

Equations (7)–(9) under initial and boundary conditions (10)–(11) have been solved by finite element method using characteristic-based split (CBS) scheme, [12,13]. The CBS scheme essentially contains three steps for semi-implicit form. In the first step, the pressure term from the momentum equation is dropped and an intermediate velocity is calculated. In the second step, the pressure is obtained from a pressure Poisson equation and finally the intermediate velocity is corrected to get the real velocity values. Any additional scalar equation such as the one about temperature and concentration can be added as the fourth step. In the present analysis, implicit solution to energy equation is added as the fourth step. The numerical calculations were carried out using a linear triangular element. A MATLAB code was used to solve the problem. The code was written using a non-dimensional form of the initial, boundary condition and the governing equations (7-11). The computations were done on uniform grids with a minimum of 21x21. Higher numbers of grids were required to achieve grid-independent results at high values of Raleigh number. A uniform linear triangular element was used to carried out the computations with non-uniform time step which was estimated according to Artificial Compressibility (AC) scheme,[13].

4. RESULTS AND DISCUSSION

In order to check the accuracy of the computer code developed in this study, the average Nusselt number (Nu) for the steady state ($\tau \to \infty$) is compared with other works. A good agreement is obtained between the present solution and the previous works as illustrated in Table (2).

Table (2): Steady state values of average Nu compared with literatures.

Ra	Ref. [4]	Ref. [8]	Present work
10^2			1.052
10^3	1.154	1.118	1.124
10^4	2.244	2.245	2.258
10^5	4.636	4.522	4.609

As a typical example the numerical results of the unsteady state natural convection in the cavity at Rayleigh number of 10^4 are shown in Fig.(2). The isotherms, velocities and the pressure contours for different time are displayed in this figure for volume fraction of (0%) and (10%). The results indicate that the isotherms and contours are qualitatively the same but they are quantitatively different. In general, the temperatures drop gradually from the hot wall to the cold wall. The temperatures of the nanofluid at (φ =10%) near the wall are always higher than the clear fluid (φ =0%) at any time. It is also found that the velocities (U,V) for the nanofluid are higher than that for the clear water whereas the pressure is less than in the nanofluid. However, up to (10%) volume fraction the effect of nanoparticles on the temperature –velocities and pressure profiles is less significant than that reported by [10].

The variation of the average Nusselt number (*Nu*) along the hot wall with time (τ) for $Ra = 10^2 - 10^5$ is presented in Fig.(3). It is found that the typical trend of *Nu*-τ relation is the same at any value of Rayleigh number, the average *Nu* decreases from the high value at the start of the heating, then reach a minimum value at the steady state. As shown in Fig.(3) the time required to reach the steady state decreases with increasing of Rayleigh number

It is indicated also that the existence of Al_2O_3 nanoparticles in the fluid decreases Nusselt number for any value of volume fraction. From the point of view of heat transfer, there are two opposite effects of addition of the nanoparticles to fluid. The first and the favorable one is the high thermal conductivity of the nanofluid and the second undesirable one is the increase in the viscosity especially at high volume fraction. This means that the heat transfer in natural convection at high Rayleigh numbers is controlled by convection while at low Rayleigh numbers is controlled by conduction. Table (3) summarizes the reduction in average Nusselt number for different values of Rayleigh number and volume fraction (φ).

In this study it found that although there is an enhancement in heat transfer due to the high thermal conductivity of nanoparticles but, such enhancement is small compared to the decline brought by the viscosity which reduces the convection currents and accordingly reduces the temperature gradient and the Nusselt number at the heated surface. This is can be verified clearly according to the temperature and the vertical velocity profiles shown in Fig.(4).

Table (3): Steady state values of average Nu for different values of volume fraction.

Ra	*Nu_{av} (Reduction %)*		
	φ=0	*φ=5*	*φ=10*
10^2	1.052	1.034 (1.7%)	1.012 (3.8%)
10^3	1.124	1.088 (3.2%)	1.062 (5.5%)
10^4	2.258	2.114 (6.4%)	1.988 (12.0%)
10^5	4.609	4.279 (7.2%)	3.905 (15.2%)

5. CONCLUSIONS

The unsteady state natural convection in a square cavity filled with nanofluid Al_2O_3-water has been studied in this work. The natural convection flow is caused by the temperature difference between the two vertical hot and cold walls, whereas both horizontal upper and lower walls are insulated. It is found that the average *Nu* decreases from the high value at the start of the heating to the minimum value at the steady state. It is also observed that the addition of Al_2O_3 nanoarticles to water always reduce the Nusselt number with a value depends on the Rayleigh number. The time required to reach the steady state is shorter for higher Rayleigh numbers and longer for lower Rayleigh numbers.

NOMENCLATURE

Cp	Specific heat (J /kg K)	T	Temperature (K)
g	Gravity acceleration (m/s^2)	t	Time (s)
Gr	Grashof number (g β(T_h-T_c) L^3/v^2)	*u*	Velocity in x direction (m/s)
k	Thermal conductivity (W /m K)	U	Dimensionless x component of velocity
L	Side length of the enclosure (m)	*v*	Velocity in y direction (m/s)
Nu	Nusselt number	V	Dimensionless y-component of velocity
p	Pressure (Pa)	*x*	Distance in x direction (m)
P	Dimensionless pressure	X	Dimensionless x coordinate
Pr	Prandtl number (μ Cp/k)	*y*	Distance in y direction (m)
Ra	Rayleigh number (Gr Pr)	Y	Dimensionless y coordinate
Greek symbols		*Subscripts*	
α	Thermal diffusivity (m^2/s)	c	cold
β	Thermal expansion coefficient (1/K)	f	fluid
θ	Nondimensional temperature	h	hot
μ	Dynamic viscosity (kg/m s)	n	nanofluid
ρ	Density (kg/m^3)	p	particle
τ	Nondimensional time	∞	environment
φ	Nanoparticle fraction		
ν	Kinematic viscosity (m^2/s)		

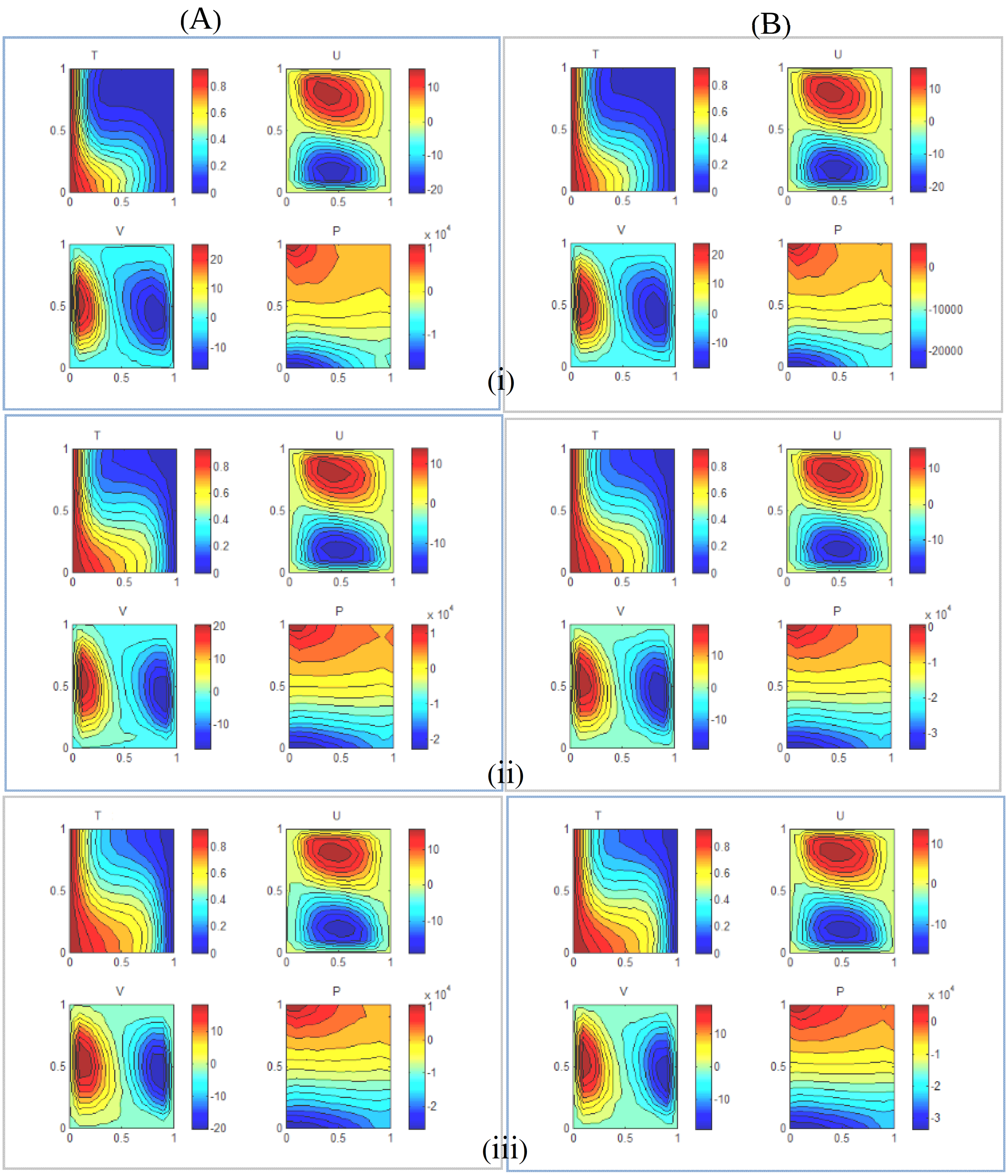

Fig.(2): Isotherms(T) and contours(U,V,P) for Al_2O_3–water nanofluid at $Ra=10^4$, ***A-left*** (φ=0%) ***B-right*** (φ=10%) at time (τ) :*i=10, ii=20, iii=30.*

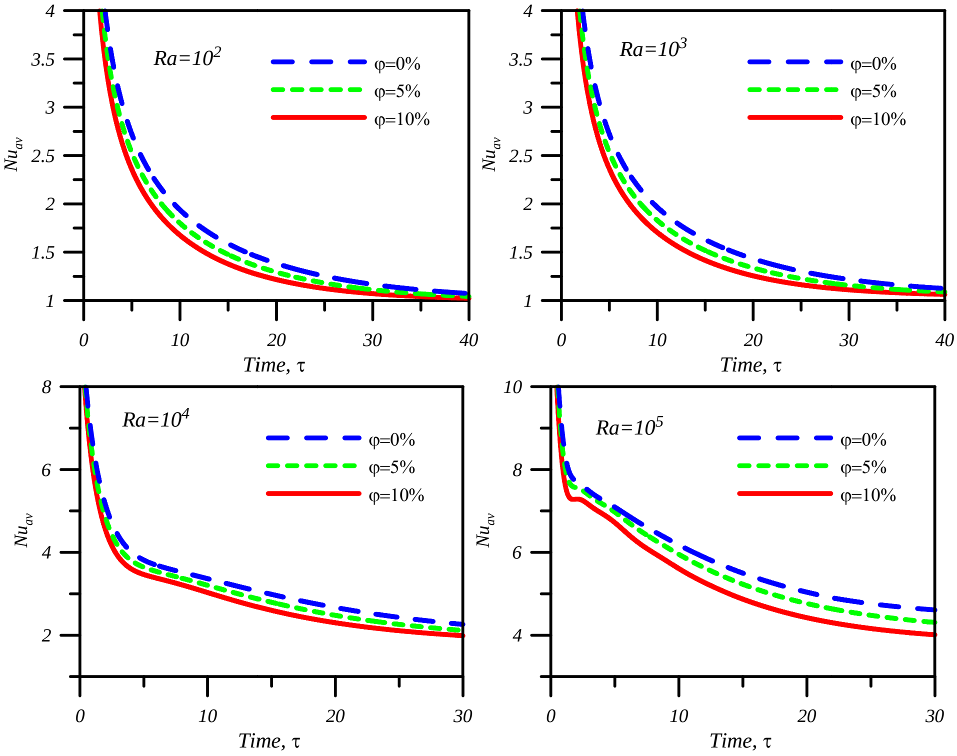

Fig.(3): Variation of average *Nusselt* number with time for different values of *Rayleigh* number.

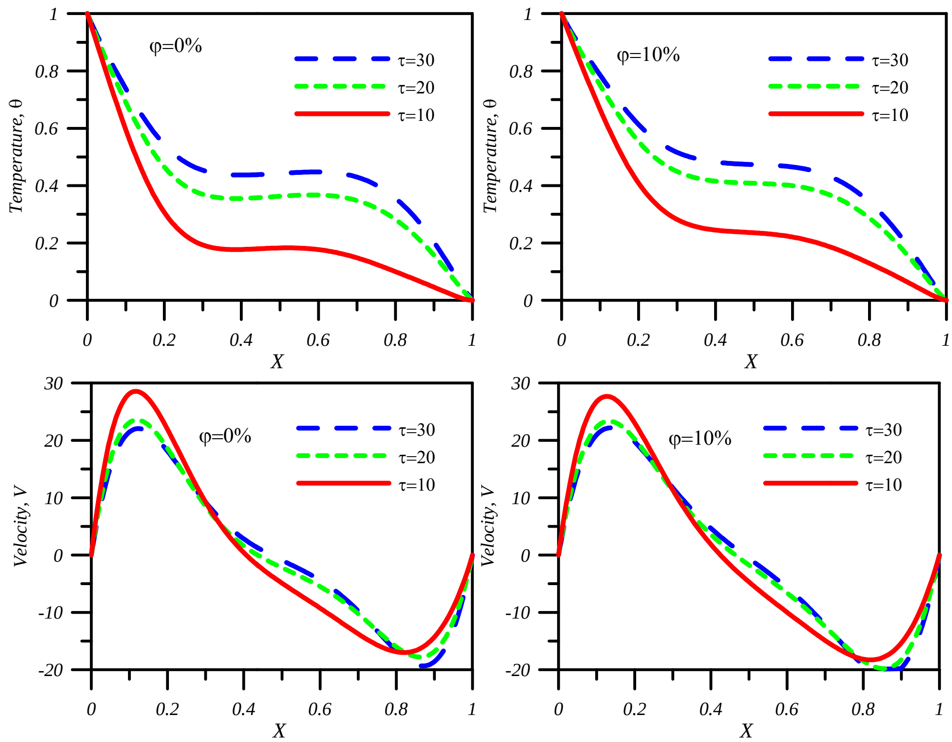

Fig.(4): Temperature and Velocity (V) profiles at (Y=0.5) for different values of time, *Ra*=10^4.

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