The Red Sea - Dead Sea Channel Project Influence upon the Dead Sea Water Balance Equation

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Abstract

At present time, water balance of the Dead Sea indicates an annual shortage reaches about one billion cubic meters of water due to high evaporation rates, industrial abstractions and lack of coming inflow. This deficit in the water balance of the Dead Sea caused a critical decline in its water level as time went by. The drop of the water surface level increased dramatically, and the estimations include different figures of annual decline range from 0.79 m per year as a long term average calculated by the authors, and 1 m per year according to Arab Potash Company (APC) records in the last decade. There were many different proposals to come over this problem, the most accepted and agreed was the construction of the Red Sea – Dead Sea conveyance channel (RDSC). Therefore, the water balance equation of the Dead Sea should include a new component expressing the inflow of the Red Sea water. Then the changes of the water column height and the related volume of water in the Dead Sea Basin (DSB) will be calculated for successive years until reaching the preferred water level of (-395m), the calculations will use the annual increment of the RDSC inflows as a positive component, where the annual shortage will be subtracted from it. The calculations will include the annual loss rate of water, the impoundment per unit time and the total impoundment end value. The calculations where conducted through two different mathematical methods, the first is the direct numerical summation method and the second is the compound numerical equations. The compound equations solving for the loss rate, the annual impoundment and the end impoundment value can be utilized to build up a mathematical model to simulate for different unknown parameters in related equation.

Introduction

Generally, the Dead Sea is a closed lake with no stream outflows which means it has only two ways of water losses, the evaporation and industrial abstraction. But, there is another important reason causes a big influence on the water balance equation of the Dead Sea, which is the deviation, withdrawal and damming of regional and local recharging tributaries in the upstream [1]. These factors endorsed the shortage in the water balance equation that is the loss of water exceeded the incoming flows. Therefore, the water level is declined continuously in a sever manner. This leads to deteriorate the water type and affects the hydrology and hydrochemistry of the water. The Dead Sea limnology indicates a hypersaline basin, which is about ten times higher than normal ocean waters. This fact is introduced into balance equation as an influencing parameter controlling the exact changes of the water column height of the DSB. That means, the real drop down of the water surface of the Dead Sea is equal to the apparent drawdown (measured) plus the thickness of the accumulated salts at the bottom of the Dead Sea. The thickness of the precipitated salts can reach more than 30% of the evaporated water height of the Dead Sea water [2].

The proposed solution for this sever decline in the level of the Dead Sea surface water is to find a perennial source of water to compensate the water balance deficit. The most applicable and approved project suggested by the World Bank is the construction of a conveyance channel connecting the Red Sea and the Dead Sea. The proposed project (RDSC) has three main different scenarios to supply the Dead Sea with certain water amounts: The first scenario is to supply one Billion Cubic Meters BCM, the second is to supply one and half BCM, and the third is to supply it with two BCM of relatively less saline water from the Red Sea, about one tenth of the Dead Sea salinity [3].

Scope of the Study

This study aims to reformulate the Dead Sea water balance equation to involve the impoundment of the Red Sea water into the Dead Sea basin. This artificial supply is due to the execution of the proposed (RDSC) project. This component will be the main influencing factor controlling the future levels of the Dead Sea water surface. This component will continue for the whole project time and it will supply the excess water in the basin until the original desired level of (-395m) is recovered.

Dead Sea Balance Equation

The general governing equation of the Dead Sea water balance can be written as follows:

Dead Sea Water Balance = ∑ Input flow – (∑ Output loss + λ) … Eq. (1)

At the equilibrium state Eq. 1 can be written as in Eq. 2, whereas the Dead Sea Water Balance will be zero.

∑ Input flow = ∑ Output loss + λ ………………………… Eq. (2)

The right hand side of the equation is more simple in the case of the Dead Sea comparing with other basins, that is this basin represent a closed lake with no natural out flows except the evaporation component.

Σ Output flow = E + IA + RV + TR + λ ……………..…. Eq. (3)

Where:

E = Evaporation,

IA = Industrial abstraction,

RV = Dams reservoir storage,

TR = Interface transgression, and

λ = Volume of precipitated salts by evaporation.

The left hand side of the equation involves all possible recharging water sources, and can be expressed as follows:

∑ Input flow = P + R + G + S + IR + RDW ………………. Eq. (4)

Where:

P = Direct precipitation,

R = Surface runoff,

G = Groundwater inflow,

S = Sewage water,

IR = Interface regression and

RDW = Red Sea water.

Mathematical methods can be used to solve the increment of the water volume in the DSB as the Red Sea water inflows into it, there are different mathematical approaches depending on different assumptions.

Mathematical Methods

The mathematical methods include the numerical summation implementing the series functions method [4], which can be used to solve for the amount of the Red Sea water (RDW) that will be charged into the DSB via the conduit channel. On the other hand, the compound equations can be used to solve varying continuous impounding with varying loss rate.

1. Numerical Summations

This cumulative amount depends on many factors, that are the initial amount of water present in the DSB at starting time of pumping $S(t_0)$, the bulk volume of impoundment X within time interval (t), and the water loss ratio (r) at the same time interval. Therefore, the following summation equation can be written as follows:

$$\mathbf{S(t) = S(t0) + (1 - r) \sum_{i=1}^{t} Xi} \qquad \text{………………..……………}\mathbf{Eq. (5),}$$

To simplify the solution of Eq. (5), we assume a constant water loss rate, and a constant inflow rate (X) at the whole time interval (t). Then, for the given above assumptions, Eq. (5) can be written as:

$$\mathbf{S(t) = S(t0) + (1 - r) * t\ X} \qquad \text{………………….. }\mathbf{Eq. (6),}$$

The simple summation of successive impoundments all over the whole period needed to reach the desired level. Table (1) shows the obtained results for the different proposed 2 impoundment scenarios, which are: X1= $1.50*10^9$ and X2 = $2.0*10^9$ m^3/year. The water loss ratio r = 70% per year, and time interval t= 5 year. The calculations for the proposed scenario of supplying one BCM of water through the RDSC is not implemented in this study as far as, it is make no big difference between the incoming value and the annual deficit in the balance equation.

Table (1): The calculated results of the impoundment scenarios in BCM for recovering the DS water level of (-395m) assuming 35 BCM total loss.

$S(t_0)$	X1	r	(1-r)	t1	S(t, x=1.5)	t2	X2	S(t, x=2)
300	1.5	0.7	0.3	1	300.45	1	2	300.6
300	1.5	0.7	0.3	5	302.25	5	2	303.0
300	1.5	0.7	0.3	10	304.50	10	2	306.0
300	1.5	0.7	0.3	15	306.75	15	2	309.0
300	1.5	0.7	0.3	20	309.00	20	2	312.0
300	1.5	0.7	0.3	25	311.25	25	2	315.0
300	1.5	0.7	0.3	30	313.50	30	2	318.0
300	1.5	0.7	0.3	35	315.75	35	2	321.0
300	1.5	0.7	0.3	40	318.00	40	2	324.0
300	1.5	0.7	0.3	45	320.25	45	2	327.0
300	1.5	0.7	0.3	50	322.50	50	2	330.0
300	1.5	0.7	0.3	55	324.75	55	2	333.0
300	1.5	0.7	0.3	60	327.00	59	2	335.4
300	1.5	0.7	0.3	65	329.25			
300	1.5	0.7	0.3	70	331.50			
300	1.5	0.7	0.3	75	333.75			
300	1.5	0.7	0.3	78	335.10			

The calculated time needed to reach the desired DS water level of (-395m) are clear in the graphic representation shown in Figure (1). It is obvious that time is increasing as drained water is decreasing in the first proposed scenario.

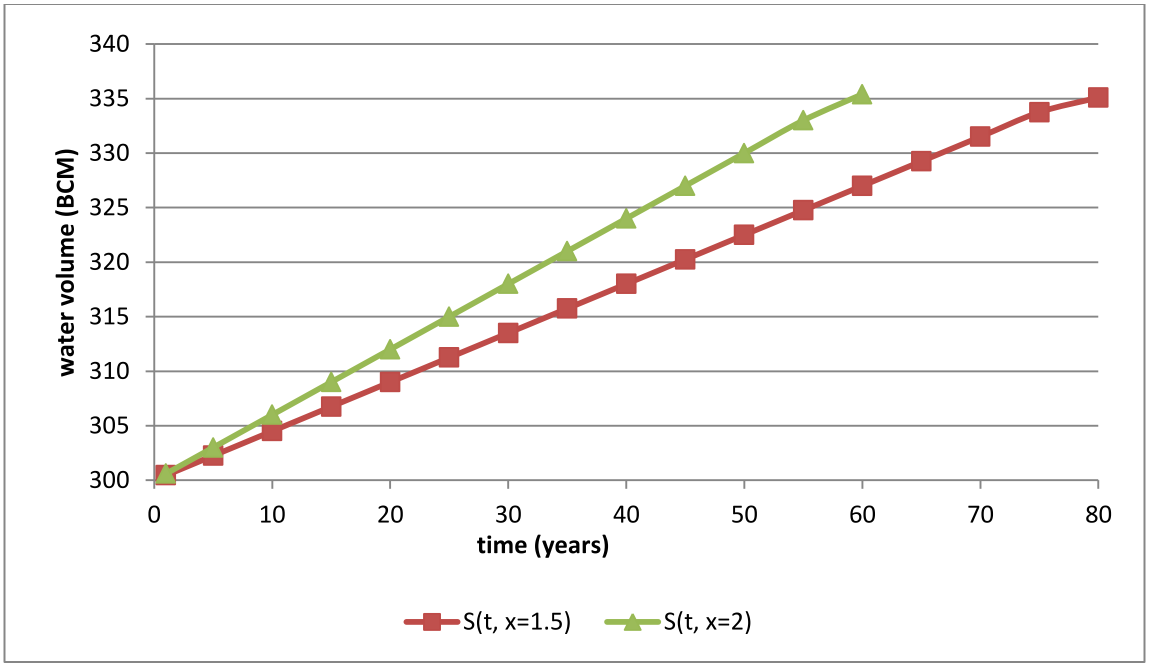

Figure (1): Comparison between the two different scenarios of the RDSC inflow.

It is obvious that the cumulative loss rate decreases as the cumulative impoundment amount increases. These calculated rates of water loss can be implemented in the following calculations to solve for the annually, monthly or daily needed impoundment amounts. The results are shown in Table 2, and the representing graph of the loss percentage with respect to time intervals of impoundment revealed a good fitness with the power function relationship (Figure 2).

Table (2): The loss % changes, during the impounded (BCM) of the Red Sea water into the Dead Sea basin.

Years	Return	Loss %	Cumulative Impoundment	Water Deficiency	Return %
0				35	
1	1	0.500	2	34	0.029
2	1	0.333	3	33	0.030
3	1	0.250	4	32	0.031
4	1	0.200	5	31	0.032
5	1	0.167	6	30	0.033
6	1	0.143	7	29	0.034
7	1	0.125	8	28	0.036
8	1	0.111	9	27	0.037
9	1	0.100	10	26	0.038
10	1	0.091	11	25	0.040
11	1	0.083	12	24	0.042
12	1	0.077	13	23	0.043
13	1	0.071	14	22	0.045
14	1	0.067	15	21	0.048
15	1	0.063	16	20	0.050
16	1	0.059	17	19	0.053
17	1	0.056	18	18	0.056
18	1	0.053	19	17	0.059
19	1	0.050	20	16	0.063
20	1	0.048	21	15	0.067
21	1	0.045	22	14	0.071
22	1	0.043	23	13	0.077
23	1	0.042	24	12	0.083
24	1	0.040	25	11	0.091
25	1	0.038	26	10	0.100
26	1	0.037	27	9	0.111
27	1	0.036	28	8	0.125
28	1	0.034	29	7	0.143
29	1	0.033	30	6	0.167
30	1	0.032	31	5	0.200
31	1	0.031	32	4	0.250
32	1	0.030	33	3	0.333
33	1	0.029	34	2	0.500
34	1	0.029	35	1	1.000
35	1	0.028	36	0	

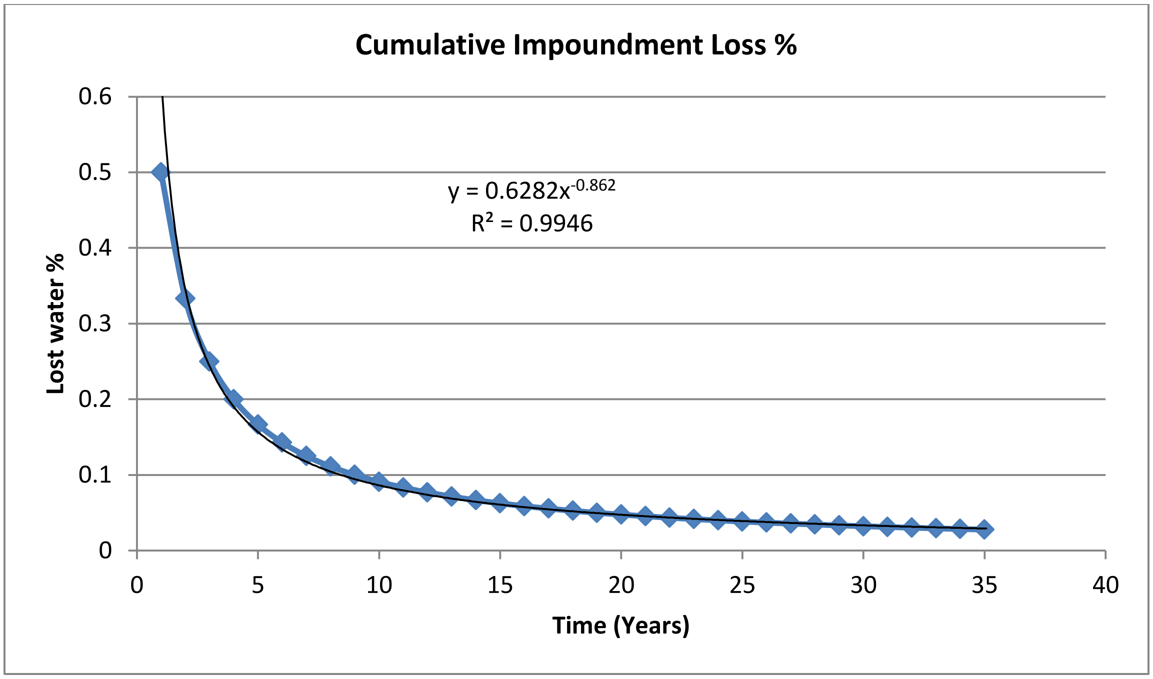

Figure (2): The loss rate percentage with respect to cumulative impoundment at the different time intervals.

2. Compound Numerical Solution:

The second method to solve for the end value of RDW incoming into the DSB through the RDSC is the compound numerical method [4, 5]. The assumed continuous compounding impoundment can be represented by different representing equations. These equations solve for the different parameters included in the governing formula that is the impoundment amounts, the loss rate and the needed periods. The rate of water loss from the DSB is an important controlling factor influencing both the impoundment amount and the related time needed to reach the desired levels.

2.1 Calculating the Loss Percentage per Period

The loss rate percentage (**r**) of the impounded waters of the Red Sea into the DSB assuming a constant continuous **Principal** inflow rate of 2 BCM per year can be calculated for the total period needed to recover the **Total** loss of 35 BCM of water within the last fifty **years** using the following formula, derived from the compound interest formula.

r =Ln ($\frac{Total}{Principal}$) /Years ………..…………….Eq. 7

Solving the above equation (7) for both scenarios of tow BCM and the one and half BCM impoundment per year can be simply done by direct substitution of the included parameters. Assume the project will continue for 50 years continuously, and the needed end recovered value is 35 BCM, then the loss rate in the first case is:

r = Ln ($\frac{35}{2}$) / 50

r = 5.7244 %

The same can be utilized for the 1.5 BCM, where the loss rate is:

r = Ln ($\frac{35}{1.5}$) / 50

r = 6.299 %

2.2 Calculating the Impoundment per Period

The general formula used to calculate the needed amount of impoundment over a certain period of time can be written as follows:

I = Total / e^{r*t} …………….Eq. 8

Where:

I = impounded amount per period,

r = the loss rate per period basis,

t = the impoundment term (i.e. number of time periods).

To calculate the annual needed impoundment amount of water in order to recover the desired original level of (-395m), the above equation (8) can be implemented. The calculations can be manipulated assuming a fixed rate of water loss through evaporation and other abstractions, throughout the whole recovering period of the desired level. The daily, monthly or annually impoundment calculations are possible to be done by the direct substitution in the given general formula above. Then, the annually deposited amount of water in the basin is calculated using the conducted loss values calculated in equation (7) for the loss rates, and assuming the total number of impoundments equal 50 years, and the total amount of deficit water to be recovered is 35 BCM, consequently the annual impoundment amount is:

I = 35 / $\boldsymbol{e^{0.0572*50}}$

I = 35 / $\boldsymbol{e^{2.86}}$

I = 35 / 17.462

I = 2.004 BCM

These calculations can be implemented for the 1.5 BCM of impounded waters, then

I = 35 / $\boldsymbol{e^{0.06299*50}}$

I = 35 / $\boldsymbol{e^{3.1459}}$

I = 35 / 23.3244

I = 1.5006 BCM

2.3 Calculating the Total Impoundment

The total impoundment (I_{total}) or the gross impoundment is the cumulative value of the loss and remain impounded waters. Assuming the periodically impoundment remains fixed throughout the recovery period. The monthly impoundment is used to recover the loss amounts plus a part of the principal deficit amount. Even though the monthly impoundment won't change, the proportion of principal and loss components will vary with each impoundment. The total volume of the water increases in the basin successively, while the loss rate decreases regarding the bulk available water.

The total impoundment throughout the 50 years is calculated by the following equation:

$$\mathbf{I_{total}} = \frac{\mathbf{t\ r\ P}}{\mathbf{1-(1+r)^{-t}}} \ \ldots\ldots\ldots\ldots\ \textbf{Eq. 9}$$

Where:

$\mathbf{I_{total}}$ = impounded amount at the end period,

P = the initial principal deficit water amount (the needed amount to be recovered)

r = the loss rate per period basis,

t = the impoundment term (i.e. number of time periods).

Therefore, the total impoundment at the end of 50 years of the project with 2 BCM annually is:

$$\mathbf{I_{total}} = \frac{\mathbf{50*0.0572*35}}{\mathbf{1-(1+0.0572)^{-50}}}$$

$\mathbf{I_{(total)}}$ **= 106.7 BCM.**

As far as the actual impounded amount of water **[$\mathbf{I_{(actual)}}$]** is equal to 35 BCM, therefore, the lost amount of the impounded waters equal to:

$\mathbf{I_{(lost)} = I_{(total)} - I_{(actual)}}$ **……………………Eq. 10**

$\mathbf{I_{(lost)}}$ **= 106.7 – 35**

$\mathbf{I_{(loss)}}$ **= 71.7 BCM**

That is the total loss percent ($\mathbf{r}_{loss}$%) is:

$$\mathbf{r_{loss}\ \% = \frac{I(loss)}{I(total)}\ \%} \qquad \text{……………………..Eq. 11}$$

$$\mathbf{r_{loss}\ \% = \frac{71.7}{106.7}\ \%}$$

$$\mathbf{r_{loss}\ \% = 67\%}$$

On the other hand, the total impoundment at the end of 50 years of the project with 1.5 BCM annually is:

$$\mathbf{I_{total} = \frac{50*0.06299*35}{1-(1+0.06299)^{-50}}}$$

$$\mathbf{I_{total} = 115.7\ BCM.}$$

As far as the actual impounded amount of water is equal to 35 BCM, therefore, the lost amount of the impounded waters equal to:

$$\mathbf{I_{total} = 115.7 - 35}$$

$$\mathbf{I(loss) = 80.7\ BCM}$$

That is the total loss percent ($\mathbf{r}_{loss}$%) is

$$\mathbf{r_{loss}\ \% = \frac{80.7}{115.7}}$$

$$\mathbf{r_{loss}\ \% = 69.7\%}$$

The annual water loss percentage is varying continuously as the cumulative impoundment progress. It is clear that the recovering cumulative impoundment amount within the proposed 50 years period time consumes about double water loss volume than the remaining water in the basin. That means the loss ratio is about 70% of the total incoming annual inflows.

Discussion

To retrieve the previous situation of the DSB at the level of (-395m). That is the level in the mid-seventies just before the separation of the deep northern DSB from the shallow southern DSB was happened [6]. The calculations dealing with the variant RDSC inflows into the DSB as a function of different proposed scenarios will be elaborated by taking the various two scenarios of transferring Red Sea water to the DSB. Thereafter, two different numerical methods where implemented in the calculations to estimate the best scenario of annual and total amount of impoundment. The first used method is the direct summation, and the second is the compound calculation method. The first method indicated that the 2 BCM impoundment scenario is better than the 1.5 annual impoundment scenario, where less total amounts of water is needed to retrieve the desired water level. The same interpretation is achieved using the compound method to calculate the needed total amount of Red Sea water impounded into the DSB. The compound calculations are based on the assumption that, during the last fifty years, the Dead Sea has lost 35 BCM of its water. This case leads to suggest that, the RDSC project can retrieve the original situation through the same period of time. If the scenario of 2 BCM of impoundment is applied to the proposed situation, then to recover the lost 35 BCM within fifty years using the calculated annual compound loss is 5.72%. It has been found that the calculated total amount of impounded waters was integrated to 106.7 BCM, where the total loss has been integrated to 71.7 BCM. These calculations indicated an average annual loss equal to 67.2 %. While for the second impoundment scenario of 1.5 BCM, the calculated annual compound loss is 6.299%. It has been found that the calculated total amount of impounded waters was integrated to 115.7 BCM, where the total loss has been integrated to 80.7 BCM. These calculations indicated an average annual loss equal to 69.7 %. Performing the comparison between the calculations based on the two different scenarios of 2 BCM and 1.5 BCM annual impoundments, assuming one BCM annual balance deficit of the DSB water, the obtained results are shown in Table (5).

Table (5): The calculated years needed to recover the DS water level of (-395m).

Inflows (I)	S(t)	r	I(loss) BCM	I(total) BCM	Years
2.000	35.000	0.05720	71.7	106.7	50
1.500	35.000	0.06299	80.7	115.7	50

Considering the scenario of one BCM of annual water inflow from RDSC project, this will play only the role of stabilizing the Dead Sea in its present situation.

The given three equations (7), (8) and (9) can perform the basic mathematical model to simulation for any scenario of impoundment including the three main variables, that is, the annual impoundment, the rate of annual water loss and the end impoundment value.

Conclusions

- The precipitated amounts of salts from the evaporated Dead Sea water are an impressive parameter controlling the water column in the DSB. This parameter (λ) should be taken into consideration and appear in the formulation of the Dead Sea water balance equation.

- In case the conveyance channel connecting both the Red Sea and the Dead Sea is executed, the artificial recharge of water to the Dead Sea should be included in the water balance equation as a special case.

- Assuming the desired 35 BCM of lost water to be recovered within 50 years, it can be found that the impoundment scenario of 2 BCM per year with 5.72 % water loss rate is better than the 1.5 BCM scenarios.

- It is important to emphasis that the water loss ratio is the most influencing factor affects the total time period needed to impound the DSB to any desired surface water level.

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