STATIC AND DYNAMIC CHARACTERISTICS OF HYDRODYNAMIC FOUR-LOBE JOURNAL BEARING WITH COUPLE STRESS LUBRICANTS

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ABSTRACT

In this work, the static and dynamic performance characteristics of four-lobe bearing operating with couple stress lubricant are presented. The modified Reynolds equation has been solved using the finite difference method. The effects of the couple stress parameter on the key performance of a four-lobe journal bearing such as; the load carrying capacity, the friction force, side leakage, the stiffness and damping, the critical mass and whirl ratio are determined. It was found that the value of the couple-stress parameter increases the load carrying capacity , decreases the friction coefficient and make this type of journal bearing more stable. The computed results show that the presence of couple stresses improves the performance characteristics of a four-lobe bearing compared to that lubricated with Newtonian fluids.

Keywords: Four-lobe bearings, stability analysis, couple-stress fluids.

1. INTRODUCTION

The problem of instability is often faced by hydrodynamic bearings operating at high speeds. The non-circular bearings are known to have better stiffness and stability characteristics. The earliest work directed towards establishing the study of the steady state performance characteristics of non-circular bearings was carried out by Pinkus [1, 2]. The stability criterion for a multi-lobe bearing was developed by Lund et al. [3] based on linearization of the Reynolds equation by small perturbation theory. Allaire, Li and Choy [4] carried out an analysis of the transient response of four multi-lobe journal bearings (elliptical, three-lobe, offset and four-lobe) subject to unbalance both below and above the linearised stability thresholds for the bearing. Pressures were measured at various locations in the four-lobe bearing by Flack et al.[5]. A pair of preloaded four-lobe bearing with flexible rotor and determined the unbalance response and instability threshold was tested experimentally by Leader et al. [6]. Static and dynamic characteristics of 6 types of multi-lobe journal bearings in turbulent flow regime have been studied by Abdul-Wahed et al. [7]. Ma and Taylor [8] presented a theoretical evaluation of five commonly used types of bearings through a comparison of their steady state

NOMENCLATURE

c	major clearance
c_m	minor clearance
$\overline{C}_{ij}$	dimensionless damping coefficients
e	eccentricity
e_p	$c - c_m$, ellipticity
h	oil film thickness
$\overline{K}_{ij}$	dimensionless stiffness coefficients
L	bearing length
l	couple-stress parameter
$\bar{l}$	dimensionless couple- stress para- meter
M	mass of journal
P	pressure
R	journal radius
W'	bearing load
W	$W'(c/R)^2/\mu_i RL$, non-dimensional bearing load
x, y, z	circumferential, radial and axial co-ordinates respectively
δ	e_p/c, ellipticity ratio
ε	*e/c*, eccentricity ratio based on major clearance
$\bar{\varepsilon}$	e/c_m, eccentricity ratio based on minor clearance
θ	angular coordinate
$\theta_{e1},\theta_{e2},\theta_{e3},\theta_{e4}$	angular coordinates at the end of bearing pads
$\theta_{s1},\theta_{s2},\theta_{s3},\theta_{s4}$	angular coordinates at the start of bearing pads
$\theta_{t1},\theta_{t2},\theta_{t3},\theta_{t4}$	angular coordinates at the trailing edges
μ	lubricant Viscosity
ν	whirl frequency
γ	whirl ratio
ϕ	attitude angle
ω	angular velocity of the journal

performance characteristics. The results show that in general the performance of the noncircular bearing is inferior to that of the circular bearing. The new type of bearing, namely, four-lobe pressure-dam bearing was studied by Mehta et a. [9].

Many practical lubrication applications may be found where the Newtonian fluid constitutive approximation is not a satisfactory engineering approach to lubrication problems. The theory of Stokes [10] is the simplest generalization of the classical theory of fluids which allows for polar effects such as the presence of a non-symmetric stress tensor and couple stresses. The couple stress may appear to a noticeable extent in the flow of liquids containing additives or in a lubricant containing long-chain molecules. The performance characteristics of hydrodynamic journal bearings using lubricants with couple stress have been studied by many researchers [11, 12]. Mokhiamer et al. [13] investigated the effects of the couple stress parameter on the static characteristics of finite journal bearings with flexible bearing linear material. Elsharkawy et al. [14] presented an inverse solution for a finite journal bearing lubricated by a couple stress fluid. A numerical study of the performance of a dynamically loaded journal bearing lubricated with couple stress fluids was given by Wang et al. [15]. Guha [16] presented the effects of couple stress fluids on the dynamic characteristics of finite journal bearings. Recently, Crosby et al.[17] studied the static and dynamic characteristics of two-lobe journal bearing lubricated with couple stress fluids.

In this paper, the four-lobe journal bearing lubricated with Newtonian and couple stress fluid have been analyzed. The effects of the couple stress on the static and dynamic characteristics have been studied in terms of the load carrying capacity, the friction force, side leakage, the stiffness and damping, the critical mass and whirl ratio.

2. THEORITICAL ANALYSIS

2.1. GOVERNING EQUATIONS

According to Stokes micro-continuum theory, a couple stress fluid is characterized by two constants μ and η whereas only one constant μ appears for a Newtonian fluid. This new material constant η responsible for the couple stress property could be determined by some experiments as discussed by Stokes [10]. With the dimension of $l = (\eta / \mu)^{1/2}$ being length, it could be considered as the characteristics length of various additives blended in Newtonian lubricant. The influence of couple stress on the system is dominated by this couple stress parameter. The modified Reynolds equation for the couple- stress fluids [17] in the non-dimensional form is given by:

$$\frac{\partial}{\partial\theta}\left(\frac{\overline{G}\left(\overline{h},\overline{l}\right)}{12}\frac{\partial\overline{P}}{\partial\theta}\right)+\left(\frac{R}{L}\right)^2\left\{\frac{\partial}{\partial\overline{z}}\left(\frac{\overline{G}\left(\overline{h},\overline{l}\right)}{12}\frac{\partial\overline{P}}{\partial\overline{z}}\right)\right\}=\frac{1}{2}\frac{\partial\overline{h}}{\partial\theta}+\frac{\partial\overline{h}}{\partial t} \qquad (1)$$

where

$$\theta = \frac{x}{R},\ \bar{z} = \frac{z}{L},\ \varepsilon = \frac{e}{c},\ \bar{P} = \frac{P(c/R)^2}{\mu\omega},\ \bar{l} = \frac{l}{c}$$

and

$$\bar{G}(\bar{h},\bar{l}) = \bar{h}^3 - 12\bar{l}^2\bar{h} + 24\bar{l}^3 \tanh\left(\frac{\bar{h}}{2\bar{l}}\right) \tag{2}$$

2.2 BEARING CONFIGURATION

The configuration of the four-lobe bearing is shown in Fig.1. The non-dimensional fluid film thickness for each lobe is given by Mehta [9]:

$$\bar{h}_i = 1 + \bar{\varepsilon}_i \cos(\theta - \phi_i) \quad i = 1, 2, 3, 4 \tag{3}$$

where $\bar{h} = \frac{h}{c}$, $\bar{\varepsilon} = \varepsilon(1-\delta)$

The eccentricity ratios of each lobe for the bearing are given by Mehta [9]**.**

$$\begin{aligned}
\varepsilon_1^2 &= \varepsilon^2 + \delta^2 - 2\varepsilon\delta \cos\left(\frac{\pi}{4} - \phi\right) \\
\varepsilon_2^2 &= \varepsilon^2 + \delta^2 - 2\varepsilon\delta \sin\left(\frac{\pi}{4} - \phi\right) \\
\varepsilon_3^2 &= \varepsilon^2 + \delta^2 - 2\varepsilon\delta \sin\left(\frac{\pi}{4} + \phi\right) \\
\varepsilon_4^2 &= \varepsilon^2 + \delta^2 - 2\varepsilon\delta \cos\left(\frac{\pi}{4} + \phi\right)
\end{aligned} \tag{4}$$

and the attitude angles of each lobe for the bearing are given by:

$$\begin{aligned}
\phi_1 &= \frac{5\pi}{4} + \sin^{-1}\left[\frac{\varepsilon}{\varepsilon_1}\sin\left(\frac{\pi}{4} - \phi\right)\right] \\
\phi_2 &= 2\pi - \sin^{-1}\left[\frac{\varepsilon}{\varepsilon_2}\cos\left(\frac{\pi}{4} + \phi\right)\right] \\
\phi_3 &= \frac{\pi}{4} - \sin^{-1}\left[\frac{\varepsilon}{\varepsilon_3}\cos\left(\frac{\pi}{4} + \phi\right)\right] \\
\phi_4 &= \frac{3\pi}{4} - \sin^{-1}\left[\frac{\varepsilon}{\varepsilon_4}\sin\left(\frac{\pi}{4} + \phi\right)\right]
\end{aligned} \tag{5}$$

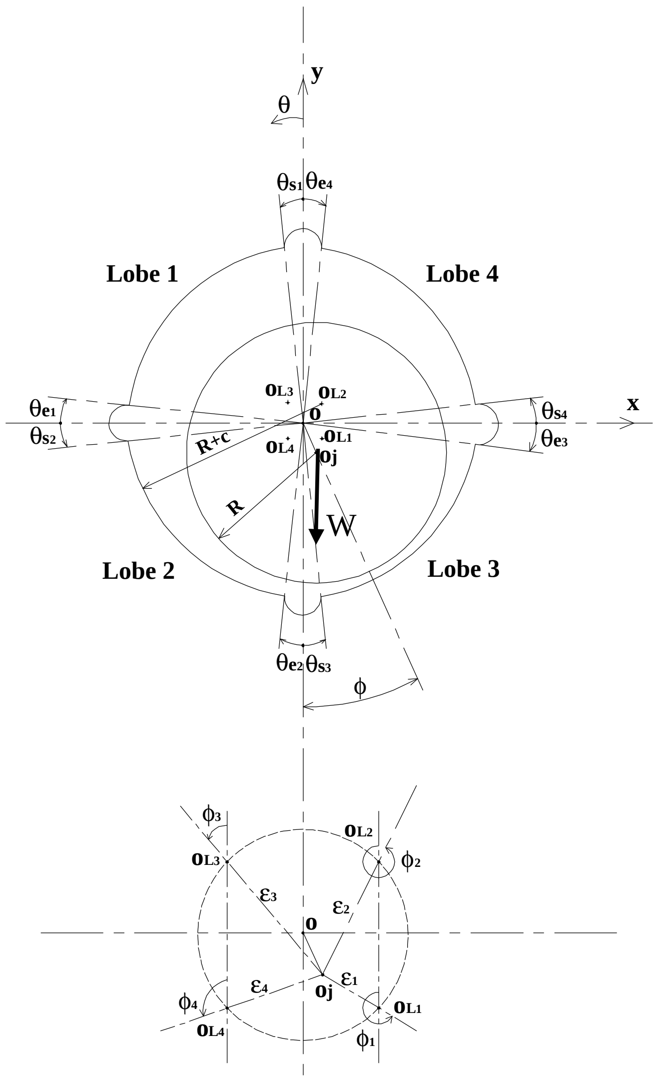

Figure 1. Bearing Geometry

2.3. BOUNDARY CONDITIONS

$\overline{P}=0$ at $\overline{z}=0$ and $\overline{z}=1$ (6a)

$\overline{P}=0$ at $\theta=\theta_{s1}$, $\theta=\theta_{s2}$ $\theta=\theta_{s3}$ and $\theta=\theta_{s4}$ (6b)

$\dfrac{\partial \overline{P}}{\partial \theta}=0$ at $\theta=\theta_{t1}$, $\theta=\theta_{t2}$, $\theta=\theta_{t3}$ and $\theta=\theta_{t4}$ (6c)

$\overline{P}=0$ for $\theta_{e1}\geq\theta\geq\theta_{t1}$, $\theta_{e2}\geq\theta\geq\theta_{t2}$, $\theta_{e3}\geq\theta\geq\theta_{t3}$ and $\theta_{e4}\geq\theta\geq\theta_{t4}$ (6d)

Equations (6c) and (6d) are the Swift-Stieber boundary conditions at the trailing edges.

2.4. STATIC CHARACTERISTICS

Equation (1) with boundary conditions were solved using iterative finite difference methods in which the value of any pressure is given by

$$A_0 \overline{P}_{i,j} + A_1 \overline{P}_{i+1,j} + A_2 \overline{P}_{i-1,j} + A_3 \overline{P}_{i,j+1} + A_4 \overline{P}_{i,j-1} = B_{i,j} \tag{7}$$

The bearing's static characteristics are obtained by solving the modified Reynolds equation (1) for static loading ($\frac{\partial h}{\partial t} = 0$). Thus, the pressure distribution, load components, and friction force could be obtained. The friction factor is

$$C_f = \frac{\int_0^{2\pi}\int_0^1 \left(\frac{1}{\overline{h}} + \frac{\overline{h}}{2}\frac{\partial \overline{P}}{\partial \theta} \right) d\theta \, d\overline{z}}{W} \tag{8}$$

The side leakage flow is:

$$\overline{Q}_s = \int_0^{2\pi} \frac{\partial \overline{P}}{\partial \overline{z}} \Big|_{\overline{z}=0} \left[\overline{h}^3 - 12\overline{l}^2 \overline{h} + 24\overline{l}^3 \tanh\left(\frac{\overline{h}}{2\overline{l}} \right) \right] d\theta \tag{9}$$

2.5. DYNAMIC CHARACTERISTICS

The fluid film stiffness and damping coefficients are respectively given by

$$\begin{pmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{pmatrix} = -\begin{Bmatrix} \partial / \partial x \\ \partial / \partial y \end{Bmatrix} \begin{bmatrix} W_x & W_y \end{bmatrix} \tag{10}$$

$$\begin{pmatrix} C_{xx} & C_{xy} \\ C_{yx} & C_{yy} \end{pmatrix} = -\begin{Bmatrix} \partial / \partial \dot{x} \\ \partial / \partial \dot{y} \end{Bmatrix} \begin{bmatrix} W_x & W_y \end{bmatrix} \tag{11}$$

by giving small values for $\dot{x}$ and $\dot{y}$ around the equilibrium position, the partial derivatives of $\dot{x}$ and $\dot{y}$ can be calculated.

2.5. STABILITY ANALYSIS

The linearized equations of the disturbed motion of the journal centre are [7]:

$$M\ddot{x} + K_{xx}x + C_{xx}\dot{x} + K_{xy}y + C_{xy}\dot{y} = 0$$
$$M\ddot{x} + K_{yx}x + C_{yx}\dot{x} + K_{yy}y + C_{yy}\dot{y} = 0 \quad (12)$$

Equations (12) are used to study the stability of the bearing system. Harmonic solution of the type:

$$x = xe^{\lambda t}\ ,\ \ y = ye^{\lambda t} \quad (13)$$

will be assumed [7] where $\lambda = \eta + i\nu$ is a complex frequency. The sign of the real part η allows the system stability to be defined. If ($\eta < 0$) the system is stable and vice versa. On the threshold of stability η =0, x and y are pure harmonic motions with a frequency $\lambda = i\nu$ .Thus equations (12) can be written as:

$$\begin{bmatrix} K_{xx} - M\nu^2 + i\nu C_{xx} & K_{xy} + i\nu C_{xy} \\ K_{yx} + i\nu C_{yx} & K_{yy} - M\nu^2 + i\nu C_{yy} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0 \quad (14)$$

For a nontrivial solution the determinant must vanish and equating the real and imaginary parts to zero gives:

$$\overline{M}\gamma^2 = \frac{\overline{C}_{xx}\overline{K}_{yy} + \overline{C}_{yy}\overline{K}_{xx} - \overline{C}_{xy}\overline{K}_{yx} - \overline{C}_{yx}\overline{K}_{xy}}{\overline{C}_{xx} + \overline{C}_{yy}} \quad (15)$$

$$\gamma^2 = \frac{\left(\overline{K}_{xx} - \overline{M}\gamma^2\right)\left(\overline{K}_{yy} - \overline{M}\gamma^2\right) - \overline{K}_{yx}\overline{K}_{xy}}{\overline{C}_{xx}\overline{C}_{yy} - \overline{C}_{xy}\overline{C}_{yx}} \quad (16)$$

where

$$\overline{C}_{ij} = C_{ij}\frac{c\omega}{W}\ ,\ \overline{K}_{ij} = K_{ij}\frac{c}{W}\ ,\ \overline{M} = \frac{Mc\omega^2}{W},\ \gamma = \frac{\nu}{\omega}$$

From equations (15) and (16), the critical mass and the whirl ratio γ are calculated. $\overline{M}_c$ is the critical mass parameter above which the bearing is unstable.

3. RESULTS AND DISCUSSION

The results are obtained for this bearing with journal radius to bearing length $\mathrm{R/L} = 1$, and couple stress parameter $\bar{l}$ ranging from 0.0, 0.2 and 0.4 ($\bar{l} = 0$ is the Newtonian lubricant case). The ellipticity ratio used in this study is 0.5.

The pressure distribution for the four-lobe bearing lubricated with Newtonian fluid is compared with the results obtained in [9] and the comparison is good.

Figure 2. depicts the pressure distribution along the circumferential at bearing mid-plane for various values of a couple stress fluid parameters. It can be seen that the pressure increases with the increase of the couple stress fluid parameter.

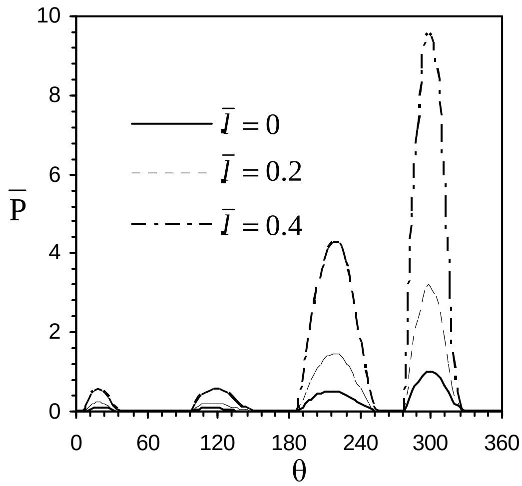

Figure 2. Pressure distribution for various values of couple stress parameter

As results of the increasing of the pressure, the load carrying capacity increases with the increase of the couple stress parameter and this can be shown in figure.3.

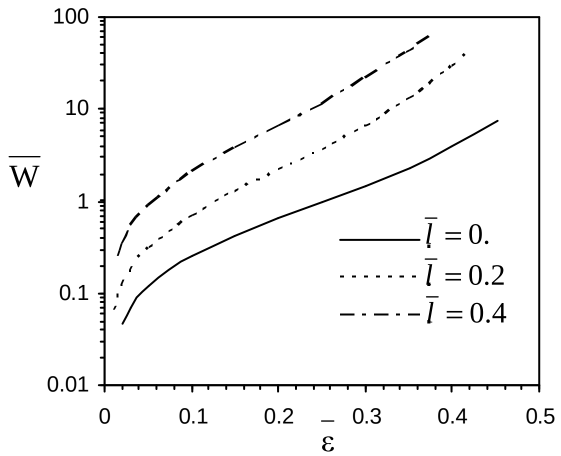

Figure 3. $\overline{W}$ versus $\bar{\varepsilon}$ for different values of $\bar{l}$

Figure 4 shows the friction coefficient versus with the eccentricity ratio for various values of the couple stress parameter. The friction coefficient deceases with increasing the couple stress parameter and its effect is more pronounced at high values of eccentricity ratio.

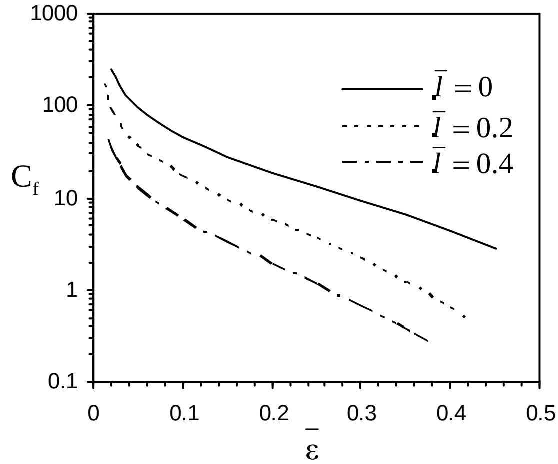

Figure 4. C_f versus ε for different values of $\bar{l}$

Figure 5 gives the variation of the dimensionless side leakage with the eccentricity ratio for different values of the couple stress parameter. The figure indicates that the effect of the couple stress parameter is not significant on the side leakage.

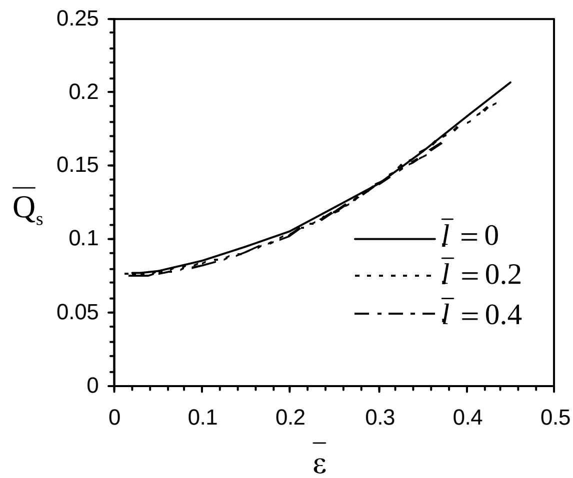

Figure 5. $\bar{Q}_s$ versus $\bar{\varepsilon}$ for different values of $\bar{l}$

Figure 6 shows the stability chart for various values of couple stress parameter. The lower and upper sides of each curve correspond to stable and unstable regions respectively. It can be seen that the stable regions for the bearings lubricated with a couple stress fluid are higher than for those lubricated with a Newtonian fluid. By increasing the couple stress parameter $\bar{l}$ the stable region increases for full range of eccentricity ratios. At high values of the eccentricity ratio the effect of the couple stress parameter $\bar{l}$ is more pronounced.

In Figure 7, the effect of couple stress parameter on the whirl ratio is shown. It is observed that the whirl ratio decreases with the increasing of the couple stress parameter and its effect is very important at higher values of eccentricity ratio.

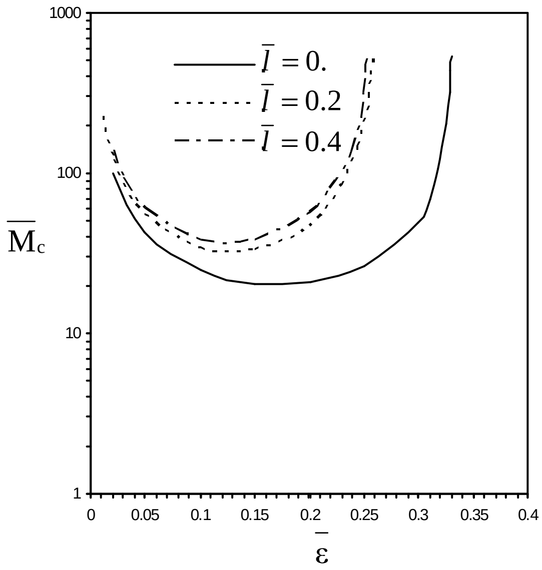

Figure 6. $\overline{M}_c$ versus ε for different values of $\bar{l}$

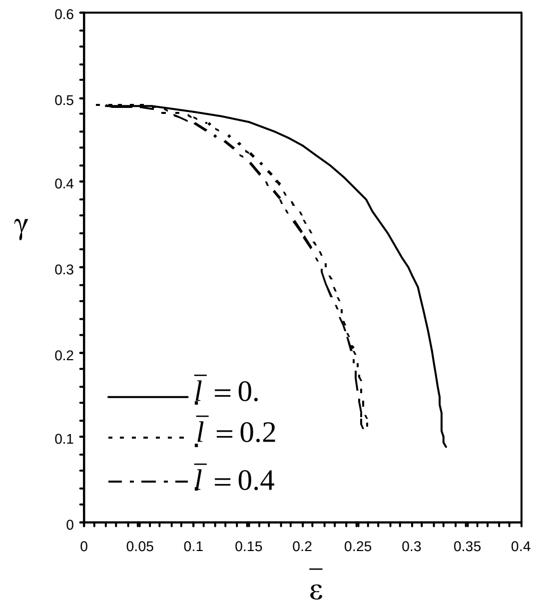

Figure 7. γ versus $\bar{\varepsilon}$ for different values of $\bar{l}$

4. CONCLUSIONS

According to the results evaluated, conclusions can be drawn as follows:

1) The effects of the couple tress parameter provide an increase in the pressure and the load carrying capacity.
2) The friction coefficient decreases with an increase of the couple stress parameter.
3) The effect of the couple stress on the side leakage is not remarkable.
4) The four-lobe journal bearing lubricated with couple stress fluid is more stable than that lubricated with Newtonian fluids.
5) The whirl ratio decreases with increasing of the couple stress parameter thus indicating more stable performance.

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