Attitude Dynamics of Axisymmetric Dual-Spin Spacecraft Containing Ring Dampers

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Abstract

This paper deals with the problem of the attitude motion of a torque free dual-spin spacecraft using ring dampers. The system composed of two balanced, axisymmetric rigid bodies and double totally filled viscous ring dampers mounted with offsets center from the spin axis on the rotor section of the spacecraft. It is shown that the fluid motion occurs in two distinct modes, previously named the nutation synchronous mode and spin synchronous mode. Using a Newton-Lagrange approach, the equations of motion are developed resulting in equations in terms of system variables and parameters. An approximate solution for the nutation angle time history is obtained and the time constants for the two modes of motion are given as a function of suitable dimensionless parameters. Comparison is then made with the solution that obtained by numerical integration of the equations of motion. It is seen that the increase in the ring center offset distance results in decreasing the time constant in the spin-synchronous mode, while it has nothing to do for the time constant in the nutation-synchronous mode.

1-Introduction

A dual spin spacecraft consists of two bodies constrained to relative rotation about a shaft connecting the bodies but otherwise free to rotate in space. The bodies are in general flexible and dissipative, as is the connection between them, and all spacecrafts are subjected to environmental torques such as the gravity gradient torque. However, as a first approximation it is useful to model dual-spin spacecraft as two rigid bodies connected by a rigid shaft and free for external torques.

Of the various schemes which have been proposed and analyzed the ones, which is called the nutation ring dampers, containing fluid in a closed tube are especially desirable since they do not involve any moving parts, other than the fluid. This type of dampers is either partially or fully filled with a viscous fluid. In this device, kinetic energy is dissipated by converted into heat when nutational motion causes the fluid to move through the tube. The first use of this damper, with the mercury fluid, was on the1958 Pioneer1 lunar [1].

A partially filled viscous ring damper on a spinning satellite was first analyzed by [2] and [3]. They assumed that the motion of the damper did not appreciably affect the precession rate of the satellite but acted only as a source of energy dissipation. With this assumption the motion of the fluid on the tube was then treated as a fluid mechanics problem. A different approach to the analysis of the mercury ring damper was taken by [4], where the fluid was modeled as a lumped mass in the presence of the assumed damping forces. They identified two distinct modes of motion, which they named, the nutation synchronous and spin synchronous modes. Alfriend [5] treated with the fluid, not as a point of mass as given in [4], but as a rigid slug of finite length. Approximate equations for time constants in the two modes have been developed and comparison with numerical integration of the exact equations shows that the approximations are good ones.

The problems of the fluid separation into several slugs and spreads out on the internal wall, which occurs in these dampers, are avoided by using fully-filled ring dampers. A completely filled viscous ring damper provided with a rigid ball, to enhance the damping performance, on dual-spin spacecraft as a passive nutation damping system was analyzed in [6], using two types of fluids, liquid mixture and neon gas. The ball motion was experienced to occur in two modes of motion as given by [4].

In this paper, double totally filled viscous ring dampers were used as a passive nutation damping system to reduce or eliminate the undesirable attitude motion of an axisymmetric dual-spin spacecraft. The dampers are mounted with equal offsets center from the spin axis in the rotor section of the spacecraft.

2-Dual-Spin Spacecraft Model

Consider the dual-spin spacecraft of Fig. (1), which consists of two symmetrical rigid bodies R_r and P constrained to rotate about the axis of symmetry with a relative angular rate $\dot{\alpha}_r$. Let a set of mutually perpendicular axes $ouvz$ and $ou'v'z'$, fixed in the center of mass of the spacecraft, such that the u and u' axes passing through the center of mass of a portion of the fluid and making an angle α counterclockwise with respect to the x-axis of the local coordinate system xyz, which is fixed in the spacecraft center of mass O.

2.1-Equations of Motion

For the spacecraft system under consideration (Fig.1), the equations of motion which describe its rotational behavior about the center of mass O in a torque free environment are obtained using the conservation of angular momentum and Lagrange's equations for the motion of the fluid inside the tubes. The angular momentum of the system about the spacecraft center of mass is given by:

$$\vec{h} = \left(A\omega_u + I_u\omega_u - I_{uz}(\omega_z + \dot{\alpha}) + I_{u'}\omega_u - I_{u'z'}(\omega_z + \dot{\alpha})\right)e_u + (A\omega_v + I_v\omega_v + I_{v'}\omega_v)e_v$$
$$+\left(C\omega_z - I_{uz}\omega_u + I_z(\omega_z + \dot{\alpha}) - I_{u'z'}\omega_u + I_{z'}(\omega_z + \dot{\alpha}) + C_P\omega_{Pz}\right)e_z \qquad (1)$$

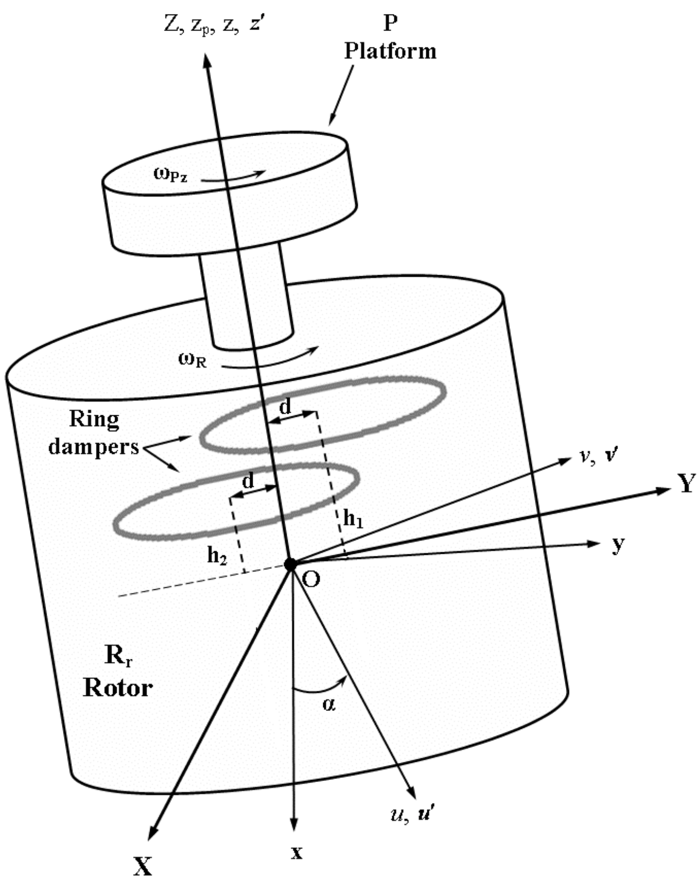

Figure (1) Dual-Spin Spacecraft Containing Nutation Dampers.

Equating the angular momentum to zero ($\dot{\vec{h}} = 0$), the equations of motion will be:

$$(A + I_u + I_{u'})\dot{\omega}_u - (I_{uz} + I_{u'z'})(\dot{\omega}_z + \ddot{\alpha}) - (\omega_z + \dot{\alpha})\left[(A + I_v + I_{v'})\right]\omega_v$$
$$+\left[C\omega_z + 2I_z(\omega_z + \dot{\alpha}) - (I_{uz} + I_{u'z'})\omega_u + C_p\omega_p\right]\omega_v = 0 \qquad (2)$$

$$(A + I_v + I_{v'})\dot{\omega}_v + (\omega_z + \dot{\alpha})\left[(A + I_u + I_{u'})\omega_u - (I_{uz} + I_{u'z'})(\omega_z + \dot{\alpha})\right]$$

$$-\left[C\omega_z + 2I_z(\omega_z+\dot{\alpha}) - (I_{uz}+I_{u'z'})\omega_u + C_p\omega_{pz}\right]\omega_u = 0 \tag{3}$$

$$C\dot{\omega}_z + 2I_z(\dot{\omega}_z+\ddot{\alpha}) - (I_{uz}+I_{u'z'})\dot{\omega}_u - \omega_v\left[(A+I_u+I_{u'})\omega_u - (I_{uz}+I_{u'z'})(\omega_z+\dot{\alpha})\right]$$
$$+\omega_u\left[(A+I_v+I_{v'})\omega_v\right] = 0 \tag{4}$$

Lagrange's equation expressed in terms of quasi-coordinates α is:

$$\frac{d}{dt}\left\{\frac{\partial T}{\partial \dot{\alpha}}\right\} - \omega_v\frac{\partial T}{\partial \omega_u} + \omega_u\frac{\partial T}{\partial \omega_v} = Q_\alpha \tag{5}$$

where, Q_α is the generalized moment associated with the quasi-coordinates α, and it is given by; $Q_\alpha = 2C_f R^2\dot{\alpha}$, C_f is the coefficient of the viscous friction between the viscous fluid and the ring wall.

After substitution the derivatives, Eq. (5) becomes:

$$2I_z(\dot{\omega}_z+\ddot{\alpha}) - (I_{uz}+I_{u'z'})\dot{\omega}_u - [(A+I_u+I_{u'})\omega_u - (I_{uz}+I_{u'z'})(\omega_z+\dot{\alpha})]\omega_v$$
$$+[(A+I_v+I_{v'})\omega_v]\omega_u = -2C_f R^2\dot{\alpha} \tag{6}$$

It is advantageous in this problem to express all the equations in suitable dimensionless variables and parameters, thus the equations of motion, Eqs.(2,3,4,6), become:

$$p' + \frac{(\lambda r - \alpha' + \lambda_s)}{D_1}q + \frac{(A_1 r + A_2\alpha' + A_3 p)}{D_1}q - \frac{A_4}{D_1}r' + \frac{A_5}{D_1}\alpha' = 0 \tag{7}$$

$$q' - \frac{(\lambda r - \alpha' + \lambda_s)}{D_2}p + \frac{(B_1 r + B_2\alpha' - B_3 p)}{D_2}p - \frac{B_4}{D_2}(r+\alpha')^2 = 0 \tag{8}$$

$$\alpha'' + C_1\alpha' + C_2 pq + C_3((r+\alpha')q - p') = 0 \tag{9}$$

$$r' - C_4\alpha' = 0 \tag{10}$$

3-Approximate Analysis (Zero Order Approximation method)

The equations of motion are strongly coupled nonlinear differential equations, an approximate solution is found depended on the zero order approximation method, which is based on the fact that the mass of the fluid is much smaller than that of the spacecraft. This means that the ratio of the fluid moment of inertia (mR^2) to the transverse moment of inertia of the spacecraft (ε) is neglected anywhere. Before developing the solution for the motion of the fluid, the equation for the nutation angle will be developed. The nutation angle θ is given by:

$$\tan\theta = \frac{h_t}{h_z} \tag{11}$$

where, h_t is the transverse component of the angular momentum, $h_t^2 = h_u^2 + h_v^2$ and h_z is the spin axis component of the angular momentum, differentiation of Eq. (11) gives

$$\theta' = -\frac{h_z'}{h_t} = \frac{ph_v - qh_u}{h_t} \tag{12}$$

Substitution of Eq. (1) for h_u and h_v gives

$$\theta' = \frac{\left[2G^2 p + G(b_1 + b_2)(r + \alpha')\right]\varepsilon q}{\sqrt{p^2 + q^2} + f(\varepsilon^2)} \tag{13}$$

where, $f(\varepsilon)$ represents the term containing the parameter ε.

Now applying the zero order approximation method, by neglecting the terms containing the parameter ε, thus the equations of motion become s:

$$r = 1 \tag{14}$$

$$p' + (\lambda_n - \alpha')q = 0 \tag{15}$$

$$q' - (\lambda_n - \alpha')p = 0 \tag{16}$$

$$\alpha'' + \frac{\eta}{\zeta}\alpha' + \frac{G^2}{\zeta}pq + \frac{G}{2\zeta}(b_1 + b_2)\left((r + \alpha')q - p'\right) = 0 \tag{17}$$

where, $\lambda_n = \sigma - 1 + \lambda_s$ is the nutation frequency. The solution for p and q is:

$$p = \omega_t \cos(\lambda_n \tau - \alpha) \tag{18}$$

$$q = \omega_t \sin(\lambda_n \tau - \alpha) \tag{19}$$

where, ω_t is the total transverse angular velocity component.

The equation for θ becomes:

$$\theta' = \left[2G^2\omega_t \cos(\lambda_n \tau - \alpha) + G(b_1 + b_2)(1 + \alpha')\right]\varepsilon \sin(\lambda_n \tau - \alpha) \tag{20}$$

Since;

$$\tan\theta = \frac{h_t}{h_z} = \frac{\omega_t}{\sigma_n + f(\varepsilon)} \tag{21}$$

where, $\sigma_n = \sigma + \lambda_s$. Now, it is need to obtain the solution for α.

4.1-Nutation Synchronous Mode

Let ϕ represent the position of the center of a portion of the fluid with respect to the nutation plane. Assuming that at τ=0, α=0, then

$$\phi = \alpha - \lambda_n \tau \tag{22}$$

Eq.(17) becomes:

$$\phi'' + \frac{\eta}{\zeta}\phi' - \left[\frac{G^2}{\zeta}\omega_t \cos(\phi) + \frac{G(b_1 + b_2)}{2\zeta}(1+\lambda_n)\right]\omega_t \sin(\phi) = -\frac{\eta}{\zeta}\lambda_n \qquad 23)$$

In the nutation synchronous mode the fluid moves at a constant rate with respect to the spacecraft, the relative position ϕ is constant. Accordingly, the particular solution of Eq. (23), is $\phi = \phi_s$. Therefore, substitution of ϕ_s into Eq. (23), and taking into account that $\phi' = \phi'' = 0$, then

$$\left[2G^2\omega_t \cos(\phi_s) + G(b_1 + b_2)(1+\lambda_n)\right]\omega_t \sin(\phi_s) = 2\eta\lambda_n \tag{24}$$

and the nutation rate equation, Eq. (20), becomes:

$$\theta_n' = -\left[2G^2\omega_t \cos\phi_s + G(b_1 + b_2)(1+\lambda_n)\right]\varepsilon \sin(\phi_s) \tag{25}$$

then, substituting the left hand side of Eq. (24) into Eq. (25), get:

$$\theta_n' = \frac{-2\eta\lambda_n\varepsilon}{\omega_t} \tag{26}$$

where, $\omega_t = \sigma_n \tan\theta$ has been used, thus the solution

$$\cos\theta = \cos\theta_o e^{\tau/\tau_n} \tag{27}$$

in which τ_n is given by:

$$\tau_n = \frac{\sigma_n}{2\eta_n\lambda_n\varepsilon} = \frac{\sigma + \lambda_s}{2\eta_n(\lambda + \lambda_s)\varepsilon} \tag{28}$$

where η_n is the damping constant in nutation-synchronous mode and given by:

$$\eta_n = \frac{8v}{r^2\Omega} \tag{29}$$

where, v is the kinetic viscosity of the fluid, Ω is the initial spin rate of the rotor.

Equation(27) is valid for $0<\theta<\frac{\pi}{2}$. For small θ the nutation angle time history can be approximated by:

$$\theta = \sqrt{\theta_\circ^2 - \frac{\tau}{\tau_n}} \tag{30}$$

At the end of the nutation-synchronous mode, the system goes into the spin-synchronous mode and the nutation angle θ_n has minimum value in this mode. So, to satisfy the condition of minimum value of the nutation angle, the angle ϕ_s should be equal to $\pm\frac{\pi}{2}$, substitute this value into Eq. (24), then the transition angle from one mode to other is

$$\tan\theta_t = \frac{2\eta_n\lambda_n}{G(b_1+b_2)\sigma_n^{\ 2}} \tag{31}$$

4.2-Spin Synchronous Mode

Substitution the solution for p and q into Eq.(17), gives

$$\alpha'' + \frac{\eta}{\zeta}\alpha' - \left[\frac{G(b_1+b_2)}{2\zeta}(1+\lambda_n) + \frac{G^2}{\varsigma}\omega_t\cos(\alpha-\lambda_n\tau)\right]\omega_t\sin(\alpha-\lambda_n\tau) = 0 \tag{32}$$

Since the spin synchronous mode occurs for the smaller nutation angle ω_t <<1, also G^2<<1, thus the last term within the brackets in Eq.(32) will be dropped.

As it is mentioned previously that the fluid, in this mode, moves with a small variation in its speed, then

$$\alpha = \alpha_\circ + \tilde{\alpha} \tag{33}$$

where, $\alpha_\circ$ is the initial value of α and $\tilde{\alpha}$ represent the small change occurs in α such that $\alpha_\circ >> \tilde{\alpha}$. The basis of this assumption is that the change in α is small compared with $\lambda_n\tau$. An approximate steady state solution of Eq. (32), may be given by

$$\tilde{\alpha} = K\tan\theta_s\left[\eta\cos(\alpha_\circ-\lambda_n\tau) - \lambda_n\zeta\sin(\alpha_\circ-\lambda_n\tau)\right] \tag{34}$$

where, θ_s referred to the nutation angle in the spin-synchronous mode, and the constant *K* is given by:

$$K = \frac{\sigma_n}{\lambda_n^{\ 2}}\left[\frac{G(b_1+b_2)(1+\lambda_n)}{2\varsigma^2\left(\lambda_n + \dfrac{\eta^2}{\varsigma^2\lambda_n}\right)}\right] \tag{35}$$

The nutation angle equation is

$$\theta' = -\left[2G^2\omega_t\cos(\alpha-\lambda_n\tau) + G(b_1+b_2)(1+\alpha')\right]\varepsilon\sin(\alpha-\lambda_n\tau) \tag{36}$$

Assuming that θ_s is small enough, so that the terms of $\theta_s^{\ 2}$ can be neglected and the change in α is small, so that $\sin(\alpha-\alpha_\circ) = (\alpha-\alpha_\circ)$ and $\cos(\alpha-\alpha_\circ) = 1$, then Eq. (36) can be reduced to;

$$\theta_s' + \theta_s\left[E_1\cos 2\lambda_n\tau + E_2\sin 2\lambda_n\tau + \frac{1}{\tau_{cs}}\right] = -G\varepsilon(b_1 + b_2)\sin(\alpha_\circ - \lambda_n\tau) \tag{37}$$

where, E_1 and E_2 are constants and the time constant in spin-synchronous mode, τ_{cs} is given by:

$$\tau_{cs} = \frac{4\varsigma^2\lambda_n\left[\lambda_n{}^2 + \frac{\eta_s{}^2}{\varsigma^2}\right]}{G^2\varepsilon(b_1 + b_2)^2(1+\lambda_n)^2\eta_s\sigma_n} \tag{38}$$

where, η_s is the damping constant in the spin-synchronous mode.

The E_1 and E_2 terms contribute nothing to the exponential decay of the solution, so that the important part of the solution of Eq.(35) is

$$\theta_s = \left[\theta_{s0} + \left(\frac{\frac{1}{\tau_{cs}}\sin(\alpha_0 - \lambda_n\tau_0) + \lambda_n\cos(\alpha_0 - \lambda_n\tau_0)}{\left(\lambda_n{}^2 + \frac{1}{\tau_{cs}{}^2}\right)}\right)G\varepsilon(b_1 + b_2)\right]e^{\frac{-(\tau-\tau_0)}{\tau_{cs}}}$$

$$- \left(\frac{\frac{1}{\tau_{cs}}\sin(\alpha_0 - \lambda_n\tau) + \lambda_n\cos(\alpha_0 - \lambda_n\tau)}{\left(\lambda_n{}^2 + \frac{1}{\tau_{cs}{}^2}\right)}\right)G\varepsilon(b_1 + b_2) \tag{39}$$

In the present study a mixture of glycerin oil with water [(75%) glycerin and (25%) water] is used as a viscous liquid because it gives minimum weight of the damper and maximum energy dissipation [6]. He calculated the damping constant in this mode by empirical relation and it is found to be equal (0.15) which is adopted in this research.

5- Results and Discussion

Figure (2) shows that the general trend of the nutation angle time history for both the analytical and numerical solutions. The effect of the second ring damper can be shown in Fig. (3), where it can be seen that the nutation angle decay in nutation-synchronous mode is decreased by (50%) compared with using only one ring damper as a passive nutation damping system. Analytically, this indicates to the fact that the number of ring damper is equal to the number of times of the time constant which it will contract.

A comparison of the time constant in nutation-synchronous mode given by Eq.(27) and an "exact" time constant is given in Figs.(4-7). The "exact" time constant is obtained by assuming exponential behavior of

$\cos\theta$ for the solution obtained by numerical integration of Eqs.(7-10) and calculating the time constant. Figures (4-7) show that the approximate solutions given by Eqs. (27) and (28) are good.

In the spin-synchronous mode, a comparison of the time constant given by Eq. (38) and an "exact" time constant is given in Figs.(8-13). The "exact" time constant was obtained by numerically integrating the exact equations of motion, assuming exponential behavior for maximum values of θ during each oscillation and calculating the time constant.

Finally, the time history of the *p, q, r* components of the spacecraft angular velocity with time are shown in Figs.(14-16). The three dimensional phase diagram with inertia ratio (1.2) is shown in Fig.(17), with initial conditions for the angular velocity components; $(p_o, q_o, r_o)^T = (0.1, 0.32, 0.98)^T$ respectively.

Table (1): Design parameters of the system.

	parameters	Value
Satellite Parameters	A	36 $kg.m^2$
	C	43.2 $kg.m^2$
	C_p	0.2 $kg.m^2$
	Ω	10.5 rad/sec
	Ω_P	0.00007272 rad/sec
	σ	1.2
Rings Parameters	R	250 mm
	r	9.5 mm
	d	50 mm
	ε	0.0009
Fluid Parameters	m	0.5186 kg
	ρ_f	1164.5 kg/m^3
	μ	$67.9\left(\frac{N.\sec}{m^2}\times 10^{-3}\right)$
	ν	58.31 mm^2/sec

The results show that the addition of second nutation ring damper on the spacecraft is greatly affecting the dynamic characteristics of the nutation damping system. Also it is seen that the increase in the ring center offset distance results in decreasing the time constant in the spin-synchronous mode, while it has nothing to do for the time constant in the nutation-synchronous mode.

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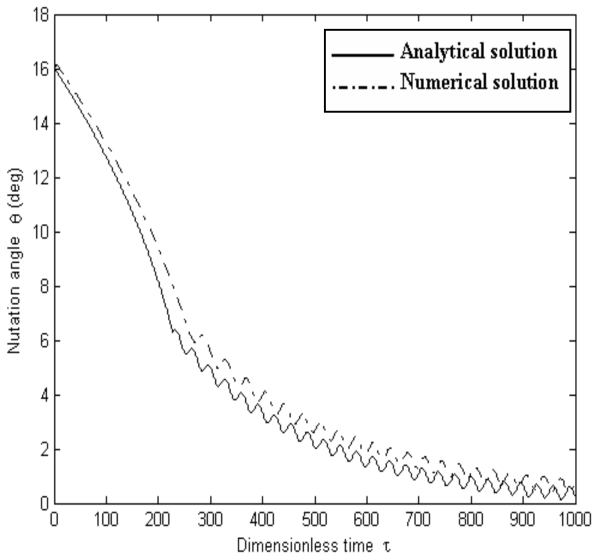

Fig. (2): Comparison between analytical and numerical solutions of the nutation angle time history.

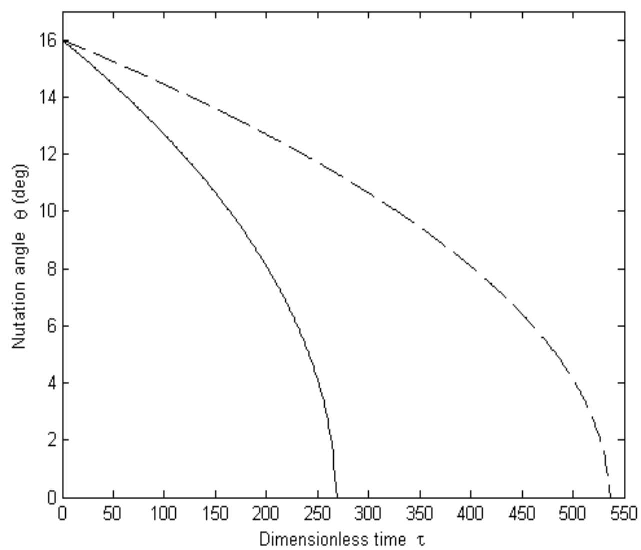

Fig. (3): Comparison of nutation angle time history of nutation-synchronous mode using single and double nutation dampers.

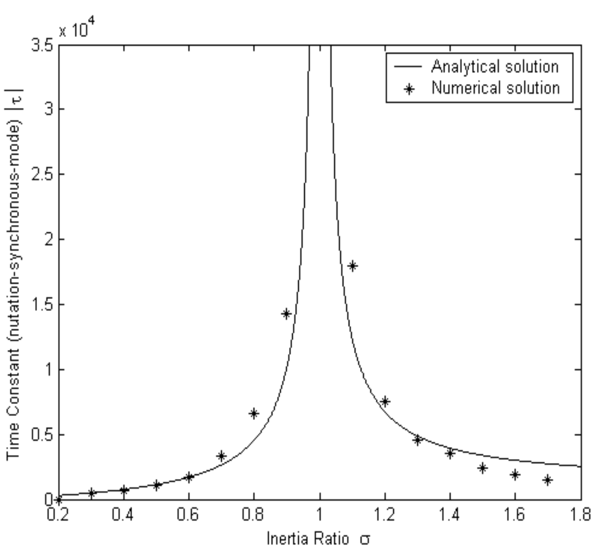

Fig. (4): The variation of the time constant of nutation synchronous mode with the inertia ratio (σ).

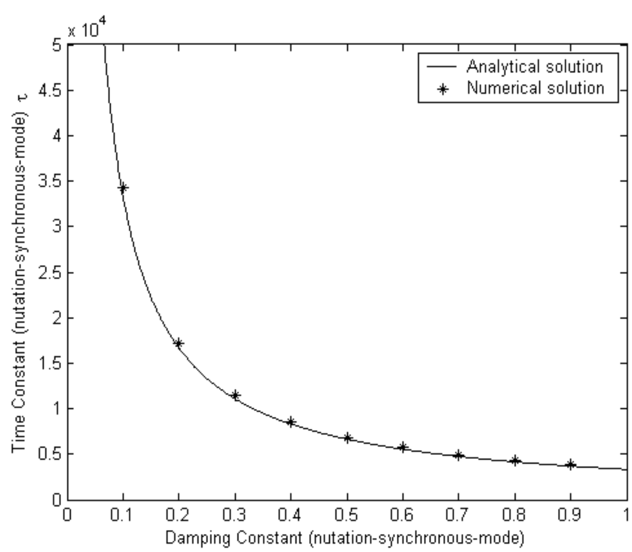

Fig. (5): The variation of the time constant with damping Constant for nutation-synchronous mode .

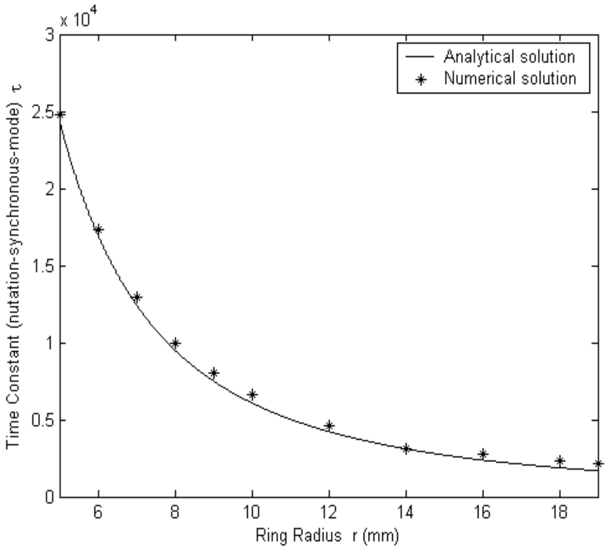

Fig. (6): The variation of the time constant of nutation synchronous mode with the ring radius.

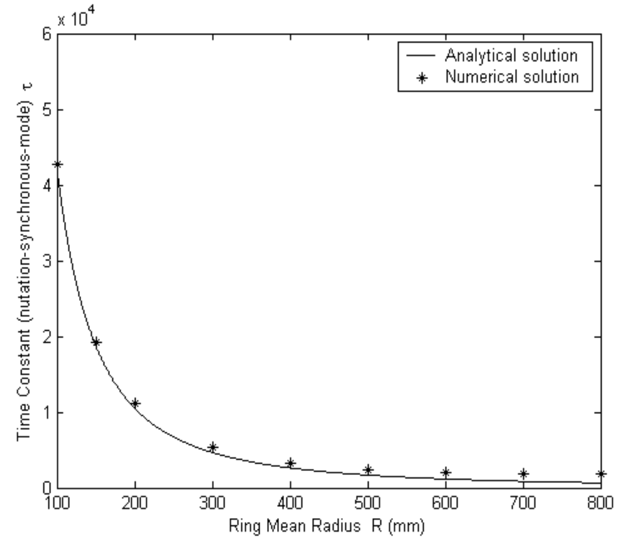

Fig. (7): The variation of the time constant of nutation synchronous mode with the ring mean radius.

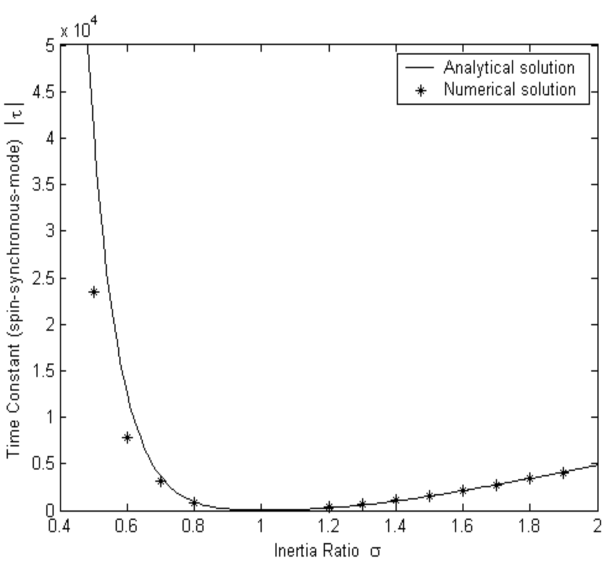

Fig. (8): The variation of the time constant of spin synchronous mode with the inertia ratio (σ).

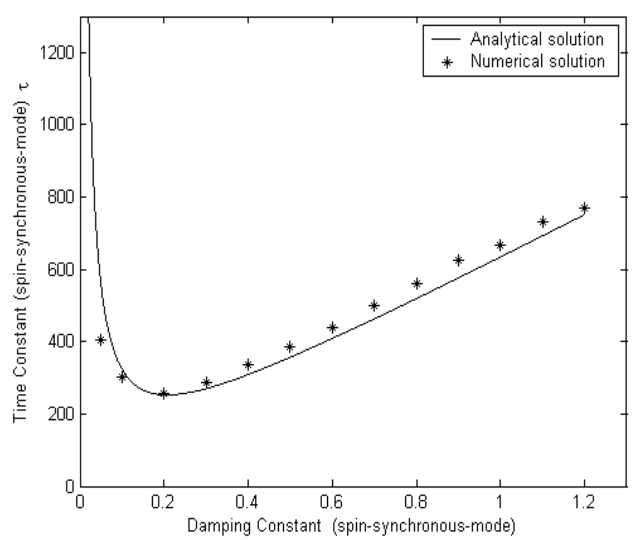

Fig. (9): The variation of the time constant with the damping Constant for spin-synchronous mode.

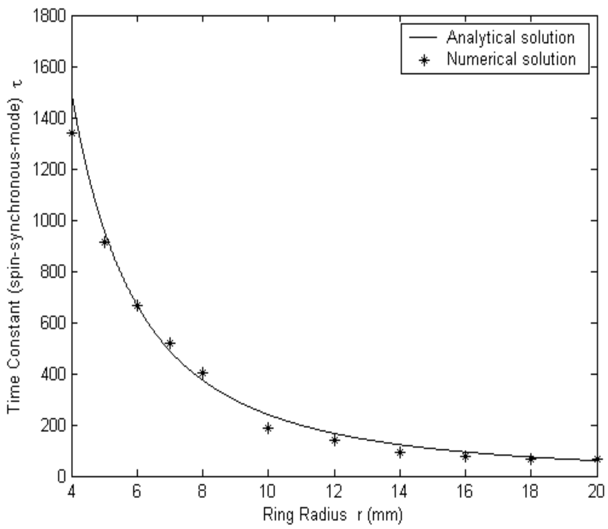

Fig. (10): The variation of the time constant of spin synchronous mode with the Ring radius.

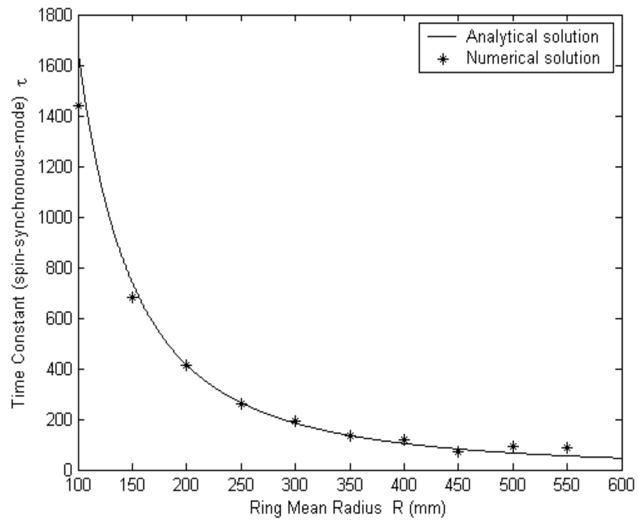

Fig. (11): The variation of the time constant of spin synchronous mode with the ring mean radius.

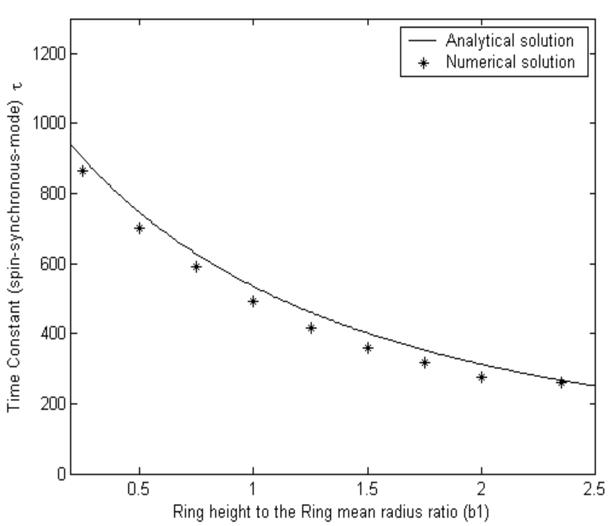

Fig. (12): The variation of the time constant of spin-synchronous mode with the ring height to the ring mean radius ratio (b_1).

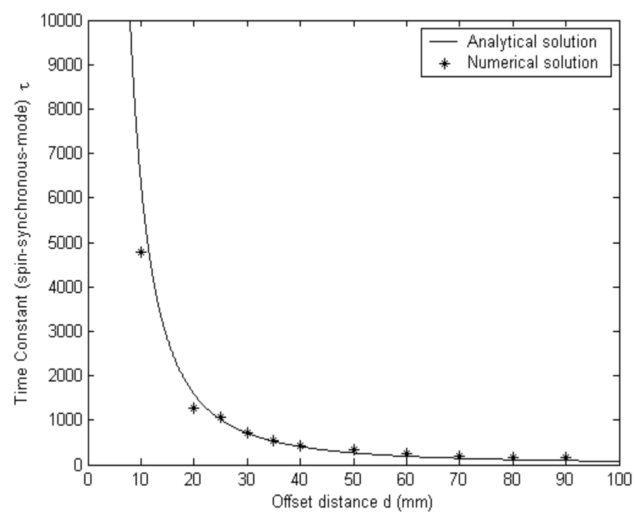

Fig. (13): The variation of the time constant of spin-synchronous mode with the ring height to the ring mean radius ratio (d).

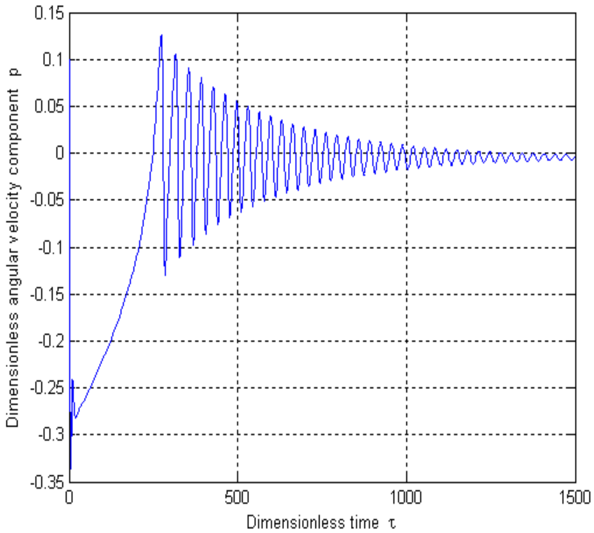

Fig. (14). The time history of the p component of the spacecraft angular velocity.

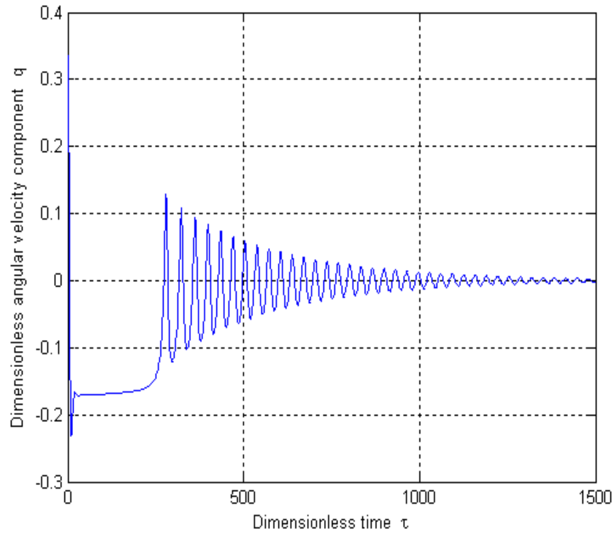

Fig. (15): The time history of the q component of the spacecraft angular velocity.

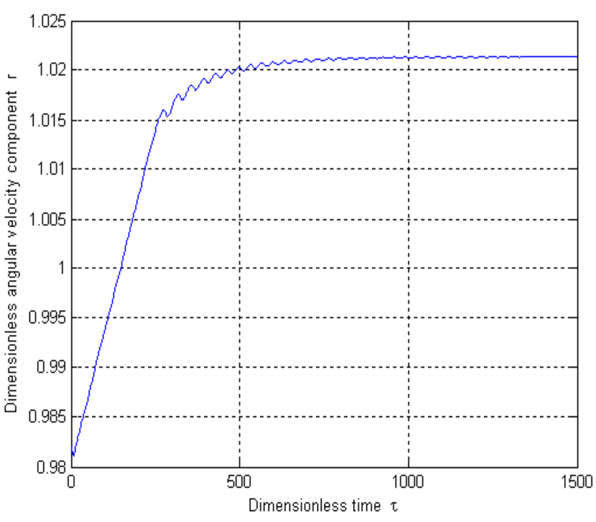

Fig. (16): The time history of the r component of the spacecraft angular velocity.

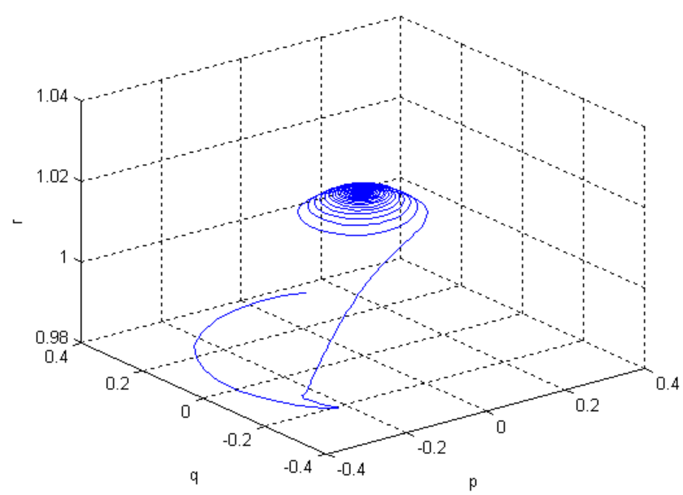

Fig. (17): Three dimensional phase diagram.