Non-Linear Analysis Of Steel Frames Subjected To Seismic Force

Wail N. Al-Rifaie, Ph.D[1] and Mohammed Yahya T. Zaindan, M.Sc[2].
[1]Professor of Civil Engineering, Philadelphia University- Amman, Jordan.
[2]Project Manager, Dijlah Establishment Construction Contracting-Amman, Jordan.

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Abstract

In this paper; non-linear analysis of steel frames subjected to seismic forces is presented.

The analysis adopts the beam-column approach. The formulation of beam-column element is is based on Eulerian approach. Changes in member chord length due to axial strain and flexural bowing are taken into account.

A modified tangent stiffness matrix which takes into consideration the geometrical non-linearity is used.

A special form to formulate the seismic forces and the monotonic load-deflection response of frames is traced using the incremental load control Newton-Raphson iterative technique.

It was concluded that considering large displacements tend to reduce the collapse loads of steel frame in the order of (4-40%) depending upon the number of stories of the frame.

Keywords: Non-linear; Steel frame; Seismic force; Earthquake duration; Sub-grade reaction.

Introduction

During the last decades, a number of major developments have taken place in the area of non-linear analysis and design. The researchers that deal with non-linear structural problems take into consideration either geometric or material non-linearity or both together.

Geometrical non-linearity occurs in two classes of non-linear problems; a stability (of which the non-linearity is mainly due to the coupling effect of axial force and flexural moment), and a large displacement problem (of which the non-linearity occurs when the deformation becomes large enough to cause significant changes in the geometry of the structure, so that the equations of equilibrium must be formulated for the deformed configuration). In comparison with linear problems, for which the terms constituting the stiffness matrix of the structure are constant, the stiffness matrix for geometrically non-linear problems contains terms that are non-linear functions of the deformations of the structure.

The objective of this paper is to derive the equation of motion, and deriving a modified tangent stiffness matrix (TSM) in global coordinates which takes into account the geometric non-linearity. The load comes from base motions due to earthquake is added to static loads.

Theory and Analytical Solution

In the beam-column approach, Eulerian, or updated lagrangian coordinates is used in developing element stiffness matrix. The effect of geometrical non-linearity is accounted for by coordinate transformation. The non-linear behavior of the element is represented by the incremental or tangent stiffness matrix.

In this paper; the beam-column element is developed by using a beam-column approach. In formulating the elemental using force-displacement relationship, the basic stiffness matrix obtained is transformed into the element stiffness matrix by coordinate transformation. Since the incremental Newton-Raphson solution technique is used, the element stiffness matrix is presented in the incremental form to facilitate the numerical implementation.

The following assumptions are considered in the modeling of the beam-column element;

1- The member is prismatic and plane sections before deformation remain plane after deformation.

2- Although large rigid body displacements are allowed, relative member deformations are considered to be small enough to justify the use of beam-column theory.

3- Influence of axial force on member stiffness is taken into account.

4- Effect of flexural moment on axial stiffness is taken into account.

5- Changes in member chord length due to axial strain and flexural bowing are taken into account.

6- Loads are applied only at joints.

1- Member Force-Deformation Relations

Considering an arbitrary prismatic member of a plane frame and let (F) and (V) denote the member and forces and displacements, respectively, in global coordinates as shown in Figure(1). In order to separate rigid body displacements from relative member deformations, an Eulerian local coordinate system is used as shown in Figure(2).

Figure (1) Member forces and deformations in global coordinate.

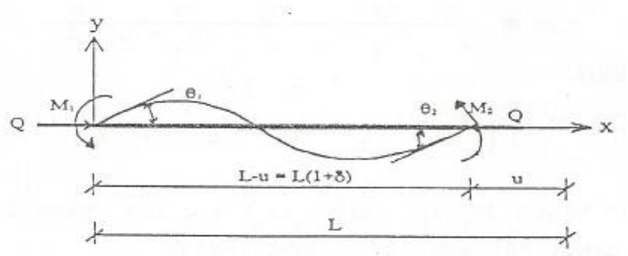


Figure (2) Relative member deformation and corresponding forces in local coordinates.

The global member forces, (F), are related to the corresponding local forces, (S) by:

$$(F) = [B]\,(S) \qquad (1)$$

In which

$$S_1 = M_1,\; S_2 = M_2,\; S_3 = Q.L \qquad (2)$$

The transformation matrix [B] is;

$$[B] = [R]\,[\overline{B}] \qquad (3)$$

From Figure(3):

$$(F) = [R]\,(\overline{F}),\; (\overline{\Delta V}) = [R]^T\,(\Delta V) \qquad (4)$$

In which;

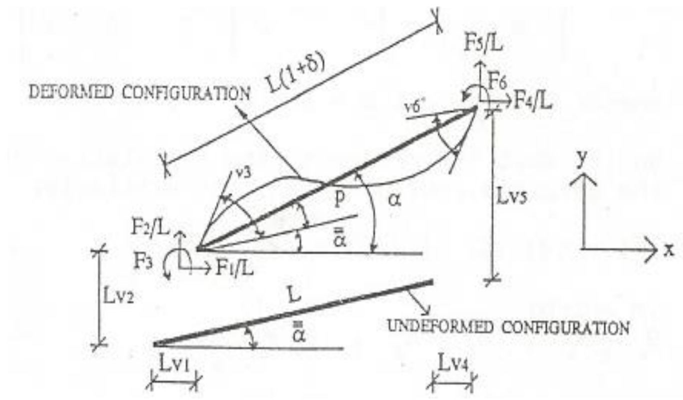
$$[R] = \begin{bmatrix} [r] & 0 \\ 0 & [r] \end{bmatrix};\quad [r] = \begin{bmatrix} C & -S & 0 \\ -S & C & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (5)$$

Where;

$$C = \cos\alpha.\; ;S = \sin\alpha. \qquad (6)$$

Noting that "α" refers to the orientation of the chord in the deformed configuration, and similarly;

$$(\overline{F}) = [\overline{B}]\,(\overline{S})\;;\; (\overline{\Delta u}) = [\overline{B}]^T(\overline{\Delta V})$$



(7)

In which;

$$\overline{S}_1 = M_1,\; \overline{S}_2 = M_2,\; \overline{S}_3 = Q.L \qquad (8)$$

$$\overline{U}_1 = \theta_1,\; \overline{U}_2 = \theta_2,\; \overline{U}_3 = U/L \qquad (9)$$

Using structural matrix analysis with;

$$\tan\alpha = \frac{(y2+L\,v5)-(y1+Lv2)}{(x2+Lv4)-(x1+Lv1)} \qquad (10)$$

and

$$L(1+s) = [\,(x_2+Lv_4-x_1-Lv_1)^2 + (y_2+Lv_5-y_1-Lv_2)^2]^{1/2} \qquad (11)$$

In which (x_1,y_1) and (x_2,y_2) are the global coordinates of joints "1" and "2" respectively.

The relationship between the relative member deformation θ_1, θ_2 and u, and the corresponding member end forces M_1,M_2 and Q, shown in Fig. (2), can be based on beam-column theory for elastic members, thus:

$M_1 = \frac{EI}{L} (C_1\, \theta_1 + C_2\, \theta_2)$ (12)

$M_2 = \frac{EI}{L} (C_2\, \theta_1 + C_1\, \theta_2)$ (13)

$Q = EA\, (\frac{u}{L} - C_b)$ (14)

In which;

C_1, C_2 = conventional elastic stability functions.

And;

$C_b = b_1\, ((\theta_1-\varphi_1) + (\theta_2-\varphi_2))^2 + b_2\, ((\theta_1-\varphi_1) + (\theta_2-\varphi_2))^2$ (15)

C_b : is the length correction factor due to bowing action.

b_1, b_2 : elastic bowing functions.

2- Member Tangent Stiffness Matrix

The incremental relationship between the member end forces and end displacements in global coordinates can be written as:

$(\Delta F) = [T]\, (\Delta V)$ (16)

In which the member tangent stiffness matrix, [T], is given by:

$[T] = [B]\, [t]\, [B]^T + \sum_{k=1}^{a} S_k\, [g^{(k)}]$ (17)

In which [t] = member Tangent Stiffness Matrix in local coordinates defined by:

$$[t] = \frac{EI}{L} \begin{bmatrix} C_1 + \frac{G_1^2}{\pi^2 H} & C_2 + \frac{G_1 G_2}{\pi^2 H} & \frac{G_1}{H} \\ C_2 + \frac{G_1 G_2}{\pi^2 H} & C_1 + \frac{G_2^2}{\pi^2 H} & \frac{G_2}{H} \\ \frac{G_1}{H} & \frac{G_2}{H} & \frac{\pi^2}{H} \end{bmatrix} \quad \text{...... (18)}$$

Where:

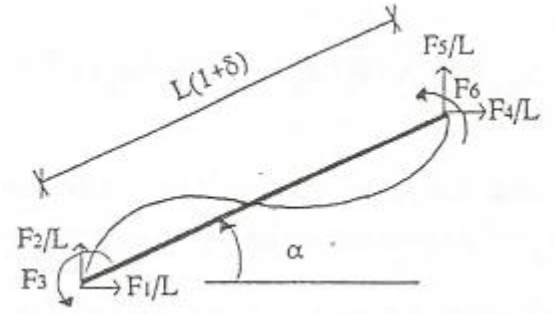
$$\theta_1 + C_2'\, \theta_2 \quad \text{......................... (19)}$$

$$\theta_1 + C_1'\, \theta_2 \quad \text{......................... (20)}$$

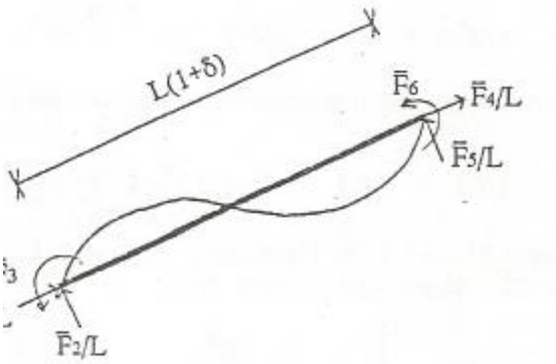
$$\text{—} + b_1'\, (\theta_1 + \theta_2)^2 + b_2'\, (\theta_1 - \theta_2)^2 \text{.............. (21)}$$

The prime superscript on C_i or b_i denotes a differentiation with respect to q.

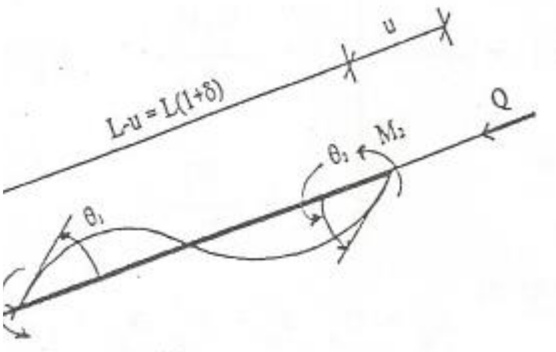
The geometric matrices, $[g^{(k)}]$ are given by[1]:



(a) Member forces in global coordinates.



(b) Member forces in member coordinates.



(c) Member forces and relative member deformations.

Figure (3) Element member forces.

In which;

$$\cdots\cdots \quad (22)$$

The prime superscript on C_i or b_i denotes a differentiation with respect to q.

The geometric matrices, $[g^{(k)}]$ are given by :

$[g^{(1)}] = [g^{(2)}]$

$$= \frac{1}{(1+\delta)^2}\begin{bmatrix} -2CS & C^2-S^2 & 0 & 2CS & -(C^2-S^2) & 0 \\ C^2-S^2 & 2CS & 0 & -(C^2-S^2) & -2CS & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 2CS & -(C^2-S^2) & 0 & -2CS & C^2-S^2 & 0 \\ -(C^2-S^2) & -2CS & 0 & C^2-S^2 & 2CS & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (23)$$

and:

$$[g^{(3)}] = \frac{1}{(1+\delta)}\begin{bmatrix} -S^2 & CS & 0 & S^2 & -CS & 0 \\ CS & -C^2 & 0 & -CS & C^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ S^2 & -CS & 0 & -S^2 & CS & 0 \\ -CS & C^2 & 0 & CS & -C^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (24)$$

3-Dynamic Analysis

In extending the method of Beam-Column approach to the dynamic case, the masses are assumed to be lumped at the joints only.

The numerical step-by-step integration technique has been used for solving the equation of motion. This technique is the only generally applicable method for analysis of arbitrary non-linear system. In addition, the technique lends itself to the solution of the base-motion response problem because of the incremental nature of the behavior associated with time dependant dynamic stresses. Within a given time step, the acceleration is assumed to be varied linearly. The deterioration of the stiffness leads to an increase in deformation, leading to the development of large secondary forces due to geometric non-linearities, and in many cases leading to instability and failure. According to that; effects excessive deformation have been introduced as P-Δ effect.

The dynamic equation of motion of the system is:

$$[M]\,(\ddot{y}) + [C]\,(\dot{y}) + [K](y) = \{f(t)\} \quad (25)$$

Where [M], [C], and [K] are mass, damping, and stiffness matrices respectively, of order (n*n). (y) is the displacement vector, $(\dot{y})$ is the velocity vector, i.e; the first derivative of (y) with respect to time (t), and, $(\ddot{y})$ is the vector of accelerations, i.e; the second derivative of (y) with respect to time (t). {f(t)} is the matrix of applied transient loads vector, n is the total DoF of the structure.

4- Modeling of Earthquakes

In the present investigation, earthquake was represented by substituting the term "F(t)" by term "$m\,\ddot{y}\,(t_i) + F(t_i)$" at initial time of earthquake (t_i). then, theses internal forces and displacements at time (t_i) and " $m\,\ddot{y}\,(t_{i+1}) + F(t_{i+1})$" at time (t_{i+1}), and so on to the end of earthquake to get the final forces and displacements.

5- Computational Techniques

To make the analysis of non-linear problems of structures more easier; two static computational techniques were used in the present investigation. They are:

1- Linear incremental method.

2- Non-linear incremental method.

Also; a dynamic computational technique was used in solving these non-linear problems which is called "The Newmark Method".

(i)***Linear Incremental Method (LIM):***

In this method, figure(4), the load is applied as a series of small increments and for each of these increments, the change in deformation is determined using a linear analysis. A so called tangent stiffness matrix, based on geometry and internal forces existing at the beginning of any step (beginning of load increment) is constructed. The total displacements and internal forces existing at the end of any step are obtained by summing the incremental changes in displacements and internal forces up to that point.

At the end of the n^{th} increment, the total applied load is given by:

$$\{P\} = \sum_{i=1}^{n}\{\Delta P\}_I \qquad (26)$$

Where $(P)_I$ is the i^{th} applied load increment vector.

Similarly, the displacements at the end of the n^{th} increment are:

$$\{X\}_n = \sum_{i=1}^{n}\{X\}_I \qquad (27)$$

The tangent stiffness for the i^{th} increment is formed for the conditions existing at the end of the previous $(i-1)^{th}$ increment. The linearised simultaneous equations to be solved in each increment is given by:

$$[\tau]_{i-1}\{\Delta X\}_I = \{\Delta P\}_i \qquad (28)$$

In which;

$$[\tau]_{i-1} = [\ \tau\ (\ \{F\}_{i-1}\ ,\ \{X\}_{i-1})\ ] \qquad (29)$$

$\{F\}_{i-1}$ is the vector of element nodal forces at the end of non-linear analysis in terms of internal forces of the members and the updated configurations of the structure from the previous load increment. However, a more elaborate method is used here in which the axial force in the members is extrapolated within the load interval according to the load increment value in order to predict a closer estimate for particular load increment. This method may be defined to be the procedure obtained from Newton-Raphson method but without any iteration.

(ii) Non-Linear Incremental Method (NLIM):

This method is similar to (LIM) in both applying load in small increments and then calculating the change in displacement caused by each load increment. The difference between the two methods is in the way of determining incremental displacement. The (LIM) employs a TSM based on internal forces and deformations existing at the beginning of the load step. A single calculation suffices to give the incremental displacement for that load step. By comparison, the NLIM is an iterative technique. For any desired load level, an approximate solution is assumed first, then an improved step-by-step via N-R type of iteration until joint equilibrium equations $\{ f_i (x_1, x_2, x_3, \ldots x_n) = p_i$, for i= 1, 2, 3, ….n is satisfied within a prescribed tolerance.

The manner in which each of the two incremental methods approximated the acual load-deformation curve is shown in Figure (4) and Figure (5).

It is shown that for LIM, numerical solution tends to drift away from the exact solution. This method requires smaller load increments than those needed when using NLIM.

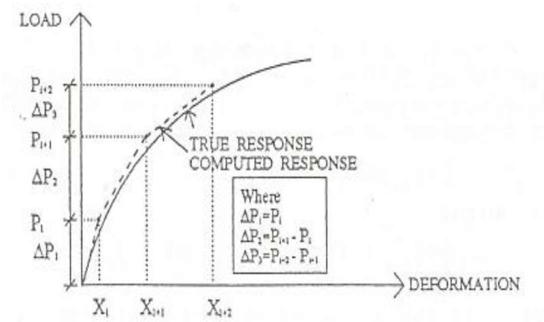


Figure (4) Linear incremental method

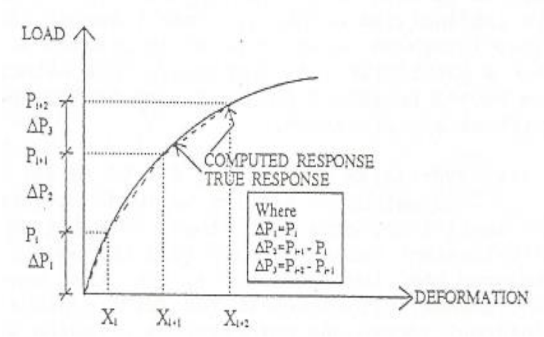


Figure (5) Non-linear incremental method

(iii) The Newmark Method

This method is the most widely used among the family of implicit methods of direct time integration for solving semi-discrete equations of motion. The Newmark method is based on the following assumptions;

$$\dot{Y}_{t+\Delta t} = \dot{Y}_t + \Delta t\ [(1-\gamma)\ \ddot{Y}_t + \gamma\ \ddot{Y}_{t+\Delta t}] \qquad (30)$$

And;

$$Y_{t+\Delta t} = Y_t + \Delta t\ \dot{Y}_t + (\Delta t)^2\ [\ (\tfrac{1}{2}\ \beta)\ \ddot{Y}_t + b\ \dot{Y}_{t+\Delta t}\ ] \qquad (31)$$

Where β and γ are constants, and they determine the stability and accuracy of the algorithm. For non-linear acceleration method (γ= 1/2) and (β=1/4). In addition to equations (30) and (31), for solutions of displacements, velocity and accelerations at time (t+Δt), the equilibrium equations of motion are also considered at time (t+Δt).

$$M\ddot{Y}_{t+\Delta t} + C\dot{Y}_{t+\Delta t} + KY_{t+\Delta t} = F_{t+\Delta t} \qquad (32)$$

Solving equation (31) for $\ddot{Y}_{t+\Delta t}$ in terms of $Y_{t+\Delta t}$ and then substituting for $\ddot{Y}_{t+\Delta t}$ in equation (30), we obtain equations for $\ddot{Y}_{t+\Delta t}$ and $\dot{Y}_{t+\Delta t}$ each in terms of the unknown displacements $Y_{t+\Delta t}$ only. Substituting these two expressions $\ddot{Y}_{t+\Delta t}$ and $\dot{Y}_{t+\Delta t}$ into equation (32) gives a system of simultaneous equations which can be solved for $Y_{t+\Delta t}$:

$$[K + \frac{\gamma}{\beta\Delta t} C + \frac{1}{\beta(\Delta t)^2} - M]\, Y_{t+\Delta t} = F_{t+\Delta t} + C \{ \frac{\gamma}{\beta\Delta t} Y_t + (\frac{\gamma}{\beta} - 1)\, \dot{Y}_t + \Delta t\, (\frac{\gamma}{2\beta} - 1)\, \ddot{Y}_t \} - M\frac{1}{\beta(\Delta t)^2} - Y_t + \frac{1}{\beta\Delta t} \dot{Y}_t + (\frac{1}{2\beta} - 1)\, \ddot{Y}_t\} \ldots \qquad (33)$$

The matrix $[K + (\gamma/b\Delta t)C + (1/b(\Delta t)^2M]$ in equation (33) is usually referred to as the " effect stiffness matrix ".

Frame Configuration and Loadings

A brief description of fifteen steel frames which are subjected to seismic loads and analyzed using the computer program (NEABF) which is programmed by the authors. These frames are divided into three groups according to its number of bays.

1- Frame Configuration

In order to insure the greatest utility of results, it was decided to limit the range of building dimensions to include building sizes most likely to be encountered in current particle.

The range of building height ranges from (1) to (10) stories at (3.46m) per story and building width at (4.0 – 6.0 m) per bay. All frames were spaced at (7.25m) centers in plane and all column bases were considered fixed.

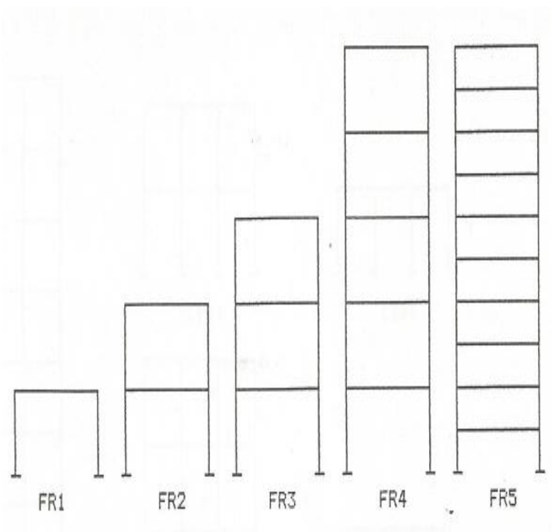
To this end, it was decided to design fifteen steel frames with some of them having a single bay or double bays while others are with three bays.

Table (1) lists the dimensions of the building frames used in this study.

Figures (6) and (7) show the configurations of the steel building frames used.

Table (1): The dimensions of the building frames used in this study

Frame No.	No. of Stories	No. of Bays	Height of Story (m)	Width of Bay (m)
FR 1	1	1	3.46	4.0
FR 2	2	1	3.46	4.0
FR 3	3	1	3.46	4.5
FR 4	5	1	3.46	5.0
FR 5	10	1	3.46	6.0
FR 6	1	2	3.46	4.0
FR 7	2	2	3.46	4.0
FR 8	3	2	3.46	4.5
FR 9	5	2	3.46	5.0
FR 10	10	2	3.46	6.0
FR 11	1	3	3.46	4.0
FR 12	2	3	3.46	4.0
FR 13	3	3	3.46	4.5
FR 14	5	3	3.46	5.0
FR 15	10	3	3.46	6.0



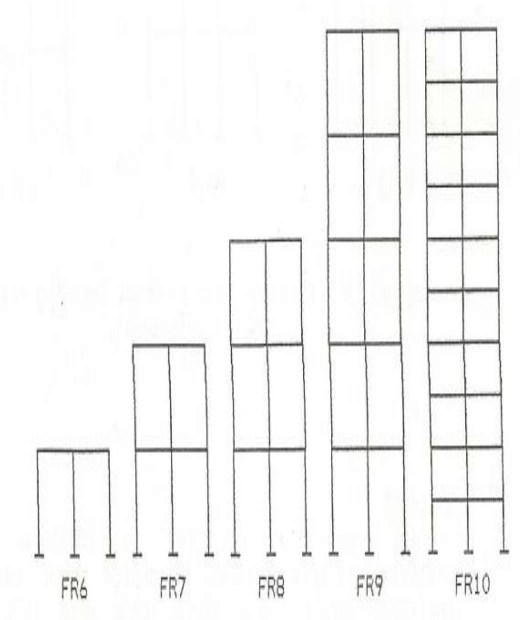


Figure (6) Configuration of steel building frames (single and double bay frames)

2- Loads

All frames were subjected to identical loading conditions and seismic loads. The design floor dead loads were (35.0 Kn/m); design live loads were (17.5 Kn/m); exterior wall loads were (6.0 Kn/m) which contribute a total of (50.0 Kn) to each exterior wall at each floor level; and design wind loads were (12.0 Kn/m).

A load combination factor of (.75) was applied to the combination of gravity and wind loads, which is consistent with the AISC specifications.

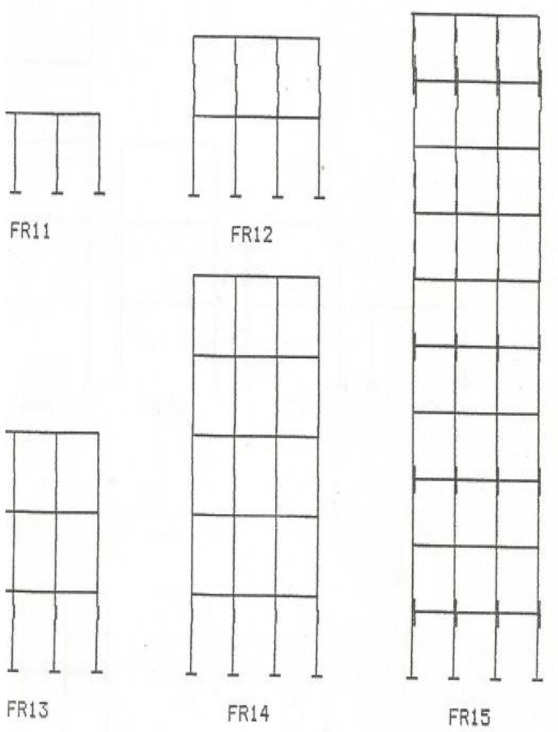


Figure (7) Configuration of steel building frames (triple-bay frames)

3- Analysis and Design of Frames

The frames listed in Table (1) were analyzed using first order elastic analysis subjected to seismic loads. Based on results from the analysis; the members of the frames were designed by selecting the appropriate wide flange shapes.

It should be noted that the steel frames mentioned above were designed using the ready computer STAAD III.

All frames have the same yield stress of 220.0 MPa (32.4Ksi) and a modulus of elasticity of $20*10^6$ MPa.

Parametric Studies

A number of chosen parameters are studied in this section.

1- Effect of large displacements on the behavior of steel frames subjected to seismic loads:

The effect of change of geometry of structure (second-order geometrical effects) on the behavior of steel frames which are subjected to seismic loads is investigated.

It should be mentioned that the term "large displacement" means that non-linear equilibrium equations must be written for the deformed configuration of the structure during loading process.

To this purpose, different steel frames were analyzed, by computer with and without considering the non-linearity.

The results of the analysis of the two types are shown in Figure(8). It may be noted from the figure shown that the horizontal displacement increases as the effect of large displacements is included in the analysis.

Figure(9) shows the effect of large displacements on the percentage of reduction in collapse load for the nine steel frames. It may be concluded that large displacement effects tends to reduce the collapse load of steel frames in the order of (4-40)% depending upon the number of stories of the frame being analyzed. This can be attributed to the structural (P-Δ) and member (P-δ) effects and also because of the other second-order geometrical effects. All these effects tend to decrease the stiffness of the system tangent stiffness matrix, and thus reduce the ultimate strength of the frame.

2- Effect of Number of stories and number of bays:

The effect of number of stories and number of bays on the fundamental period of structures which are a function of many parameters is presented in this section. All fifteen frames have been analyzed with considering fixed end conditions. The results are shown in Figure(10). From the figure, it can be concluded that the fundamental period increase with increasing the number of stories and number of bays.

Figure (11) shows the effects of number of stories and number of bays on the horizontal displacements. It can be observed that the horizontal displacements increase with increasing the number of stories, and decrease with increasing the number of bays.

3-Effect of time step size:

In this section, the influence of the time step size on stability and accuracy of the solution is presented. For that, a series of tests were made using various time step intervals (0.0001, 0.00025, 0.005 and 0.001 seconds). These time step intervals were chosen to give reasonable practical range of stepping. The influence of the time size on the horizontal displacements of the frame is shown in Figure (12), and the effect of the time step size on the axial force of the frame is shown in Figure (13). These two figures show that the time step size has a significant effect on the solution.

From the figures, it can be concluded that the smaller the time step the stable and closer the results, and using time step equal to (0.001) gives both economical and accurate resultsand decreasing time step bellow this value may not lead to significant or justified increased in accuracy.

4-Effect of static loads:

Static loads are almost associated with mass, which placed on or in the building. This additional mass can be beneficial to the structure subjected to earthquake loading, because it increases the inertial which opposes the motion of the structure. The effects of static loads are included by computing the static displacements as initial displacements for the transient solution and by applying both static and dynamic loads during the transient solution. It can be seen from figure (14) that including the static force in the analysis has negligible effects. Static loads are often ignored on earthquake resistant structures because they are usually small relative to the loads produced by an earthquake.

5-Effect of subgrade reaction (K_a):

The effect of subgrade reaction on the displacements and forces of the super-structure was investigated by considering different values of subgrade reaction as (4KN/m^2), (10MN/m^2), (100MN/m^2) and rigid support (i.e., K=∞). The influence of subgrade reaction on the horizontal displacements of the structure is shown in Figure (15). These results indicate that the values of the horizontal displacements decrease when the subgrade reaction's value increases.

6-Effect of Earthquake duration:

In this section, the effect of earthquake duration on the behavior of the structures is presented. The results are shown in Figure (16) three durations are considered in the analysis of different frames, it may be concluded that the earthquake duration has a small effect on the behavior of the structure, especially, with earthquakes which have long durations.

Conclusions

Based on the results obtained in the present paper, several conclusions may be drawn. These may be summarized as follows:

1- This investigation shows that large displacement in elastic behavior of plane steel frames subjected to earthquake can be accurately predicted using the beam-column approach.

2-Considering the large displacements in the analysis decreases the ultimate strength of the structure by a percentage of about (10%).

3-The fundamental period increases with increasing number of stories and number of bays, and increasing number of stories tends to increase the horizontal displacement, while the fundamental period decreases as the number of bays increase.

4-The influence of existing static loads during the analysis of steel frame structures subjected to seismic forces is small during the earthquake duration.

5-table and accurate results in the dynamic analysis are affected by the selection of the time step size.

6-The dynamic response of steel frame structure is substantially affected by the values of sub-grade reaction.

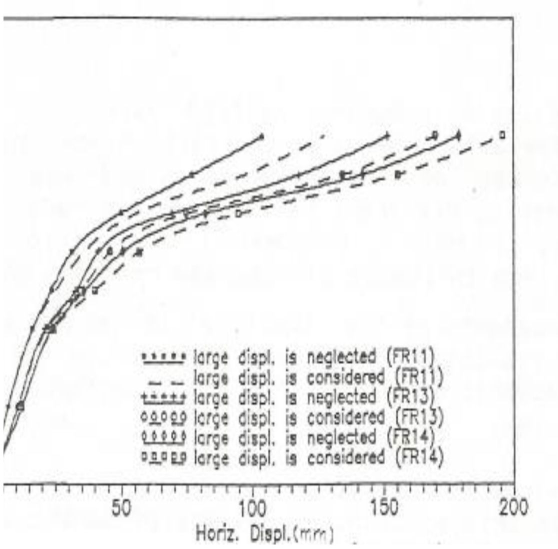


Figure (8) Effect of large displacements on load-deflection behavior for the steel frames (FR11,FR13,FR14)

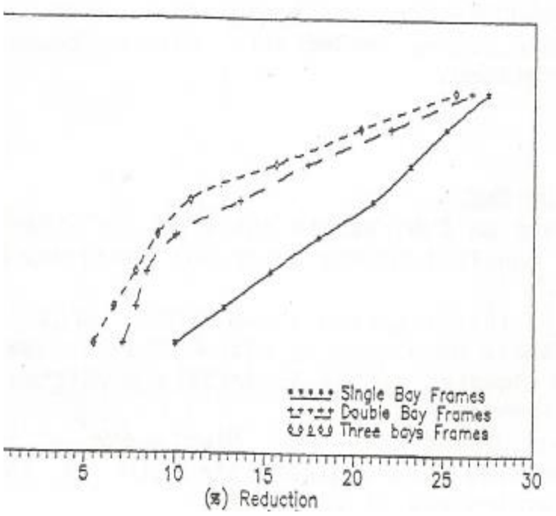


Figure (9) Effect of large displacements on (%) reduction in collapse load for fifteen steel frames

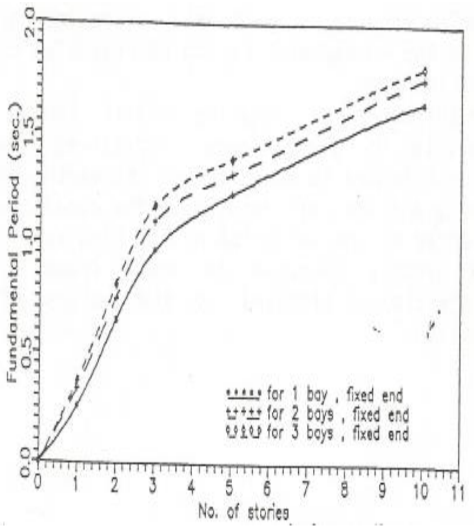


Figure (10) Effect of number of stories on the fundamental period of different number of bays

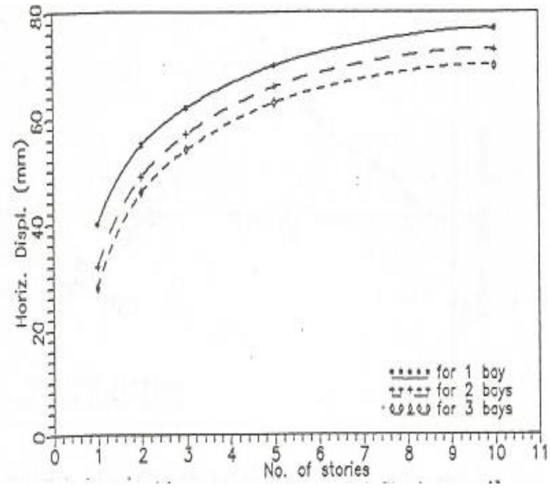


Figure (11) Effect of number of stories on the horizontal displacement of different number of bays

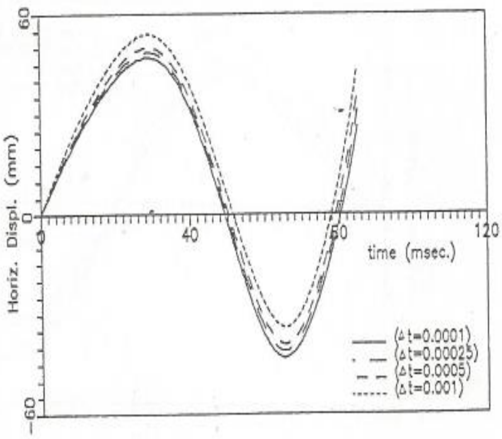


Figure (12) Effect of time step on the horizontal displacement of the frames

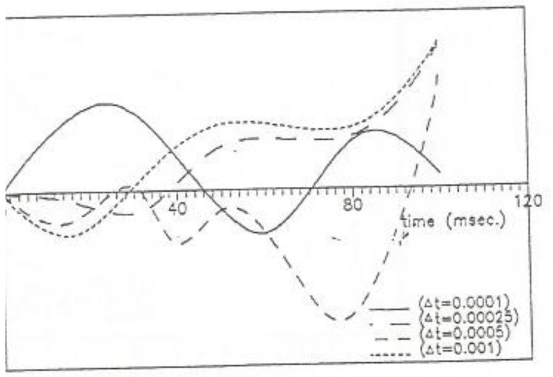


Figure (13) Effect of time step on the axial force of the frames

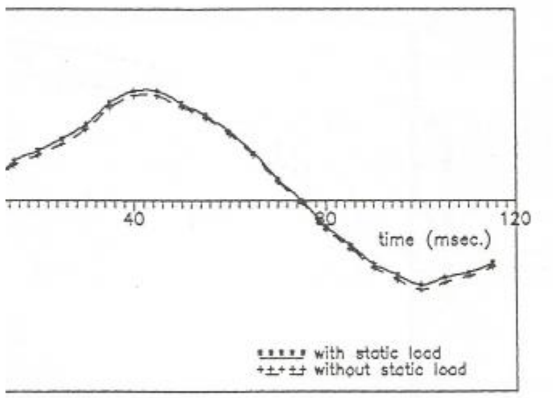


Figure (14) Effect of static loads on the horizontal displacements of the frames

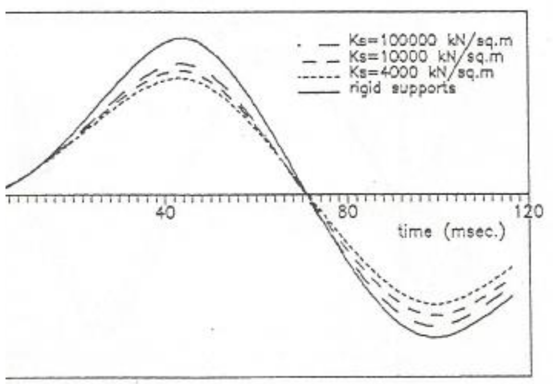


Figure (15) Effect of sub-grade reaction (K_s) on the horizontal displacement of the frames

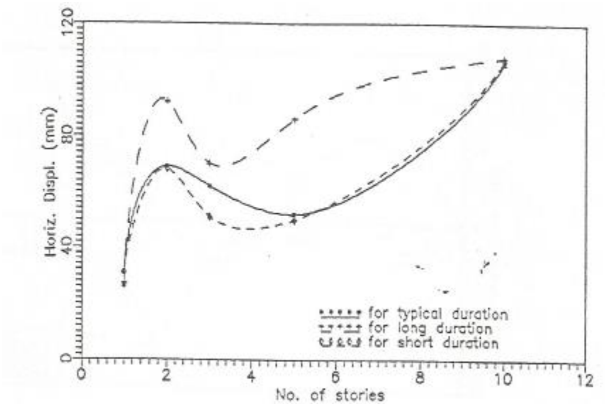


Figure (16) Effect of earthquake duration on the horizontal displacement of the frames

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