Heat Transfer Analysis of a Space Radiating Fin with Variable Thermal Conductivity

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ABSTRACT

Efficient heat removal systems are a prerequisite for the safe and satisfactory operation of a satellite. Application of fins such as in space radiating fins represents an important part of the satellite thermal control system. Consideration of constant thermo-physical properties, such as thermal conductivity, may be helpful for the evaluation of fin's temperature distribution. However, a temperature dependent thermal conductivity is considered presently for when there is a large temperature difference. The finite volume method is employed to simulate numerically the temperature distribution in a space radiating fin. The present results are compared with those of two other analytical methods and good agreement is shown.

Keywords: Finite volume method, Heat transfer, Space radiating fin, Variable thermal conductivity

1. INTRODUCTION

Satellites are playing an increasingly important role in the quality of human's life. Every Satellite has different subsystems which control its performance. One of these subsystems is the thermal control unit devised to lose the generating heat that is produced in other sections, particularly, in the electronic subsystem. Considering the space medium, the heat transfer modes are solely conduction and thermal radiation. The main role in the satellite heat transfer is carried out by radiation. Usually, space radiating fins are used for performing the heat loss by radiation. One of the main objectives of a thermal design is to maintain the temperature of a heat dissipating component at or below a specified value.

The majority of works reported on space radiating fins, including references [1] - [4], is on the basis of the constant thermal conductivity of the fin material. Bartas and Sellers [1] have studied a heat rejecting system consisting of parallel tubes joined by web plates that serve or served (because of joined) as extended surfaces. Stockman et al. [2] have compared one-dimensional and two-dimensional analyses in the central fin-tube radiators and have shown good agreement between one-dimensional and two-dimensional analyses. Cockfield [3] has discussed the role of the radiator as a structural element in the spacecraft applications. Naumann [4] has investigated analytically/numerically the thermal design of heat pipe/fin type space radiators for the case of uniformly tapered fins as well as for flat fins with constant thermal conductivity assumption. Whereas, the temperature difference between the fin base and its tip is very high in the actual situation. Hence, the variation of material's thermal conductivity should be taken into consideration.

Arslanturk [5] has evaluated the temperature distribution along a radiating fin by the analytical ADM[1] method. He has assumed a temperature dependent thermal conductivity of fin material and compared his results with those of Naumann [4] and the agreement is shown to be satisfactory. The Adomian decomposition method provides an analytical solution in terms of an infinite power series. Also, Hosseini and Ghanbarpour [6] have applied the HPM[2] method for a variable thermal conductivity, displaying their results to be in good agreement with those of reference [5]. The HPM is a new method to solve a non-linear differential equation analytically and it is on the basis of the perturbation technique.

The present heat transfer analysis of a space radiating fin is carried out numerically by developing a computer code which is based on the finite volume method [7]. The governing equation for the one-dimensional configuration is the energy equation encompassing the conduction and thermal radiation modes, under the steady-state condition. The thermal conductivity of fin material is assumed as a linear function of temperature. Also, all radiating surfaces are considered as diffuse and gray. The contact resistance is regarded as negligible. The results on the temperature distribution are obtained for different geometric and thermal variables.

2. PROBLEM DEFINITION

The configuration of present radiating fin is shown in Fig. 1. It is based on an optimized shape provided by Naumann [4] and Arslanturk [5]. The fin material is considered as Aluminum (Al) It is assumed that the fin has a length, w and thickness, D connected to a tube at its base which acts as a heat pipe. All of the geometrical parameters are measured in meter. The fin base at $x = 0$ is held at constant temperature, T_b and both side surfaces of fin can radiate to outer vacuum space, which is considered at absolute zero temperature. There are no gradients across the thickness of fin and no significant radiation from the edges, because the thickness is assumed to be thin enough.

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[1] Adomian Decomposition Method
[2] Homotopy Perturbation Method

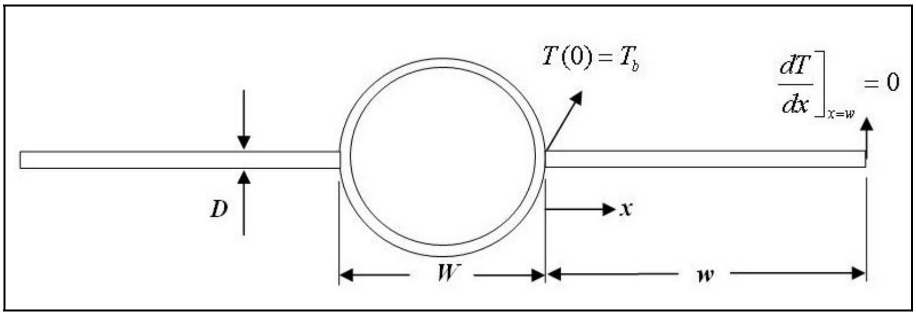

Fig.1. Fin configuration

Also, it is presumed that the fin has diffuse and gray surfaces with a constant emissivity (ε). Radiation exchange between surfaces of pipe and fin is negligible. Assuming $D << w$, the problem is solved as one-dimensional heat flow (in the x direction). The governing energy equation is as follows:

$$D \frac{d}{dx}\left[k(T)\frac{dT}{dx}\right] = 2\varepsilon\sigma T(x)^4 \tag{1}$$

where, $T(K)$ is temperature, σ is Stefan-Boltzmann constant and $k(W/m.K)$ is the thermal conductivity of fin material which is assumed to depend on the temperature linearly [5]:

$$k(T) = k_b[1 + \lambda(T - T_b)] \tag{2}$$

where, k_b is thermal conductivity at fin base and $\lambda(1/K)$ is slope of temperature-conductivity curve. The boundary conditions for the present fin geometry are defined as in equation (3):

$$\begin{aligned} \frac{dT}{dx} &= 0 \qquad at \qquad x = w \\ T &= T_b \qquad at \qquad x = 0 \end{aligned} \tag{3}$$

3. NUMERICAL DETAILS

The present problem is aimed at the prediction of temperature distribution in an extended surface while combined conduction-radiation heat transfer is taking place. One-dimensional conduction across the fin and radiation heat loss from side surfaces are taken into account. The governing equation for the present computation is considered as follows [7]:

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right) + S = 0 \tag{4}$$

where, S is radiation heat loss that has a negative sign. The discretization procedure, based on the finite volume technique [7], is carried out on the above equation to solve the problem numerically.

To derive the discretization equation, the grid point cluster shown in Fig. 2 is employed. The main point is P, where the temperature is to be determined. This grid point has the grid points E and W as its east and west neighbors, respectively. The narrow line shows the face of the control volume while letters e and w denote these faces. For the one-dimensional problem under consideration, the thickness in the y and z direction is assumed as unity. The grid points are distributed in x-direction uniformly.

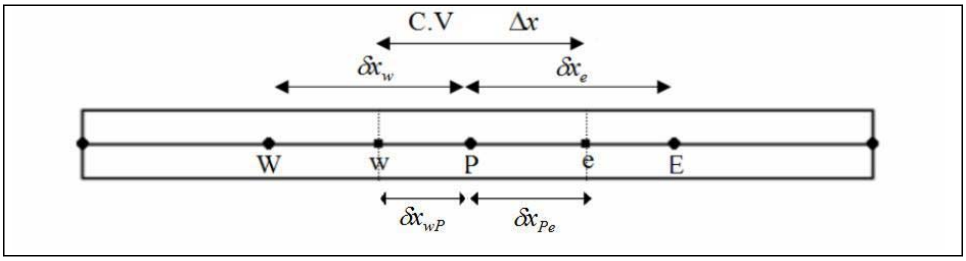

Fig.2 Grid point cluster for the one-dimensional problem

Hence, equation (5) is resulted from the integration of equation (4):

$$\left(k\frac{dT}{dx}\right)_e - \left(k\frac{dT}{dx}\right)_w + \int_w^e S dx = 0 \tag{5}$$

The derivatives of dT/dx in equation (5) are evaluated from piecewise linear profile. Then, the resulting equation is as follows:

$$\frac{k_e(T_E - T_P)}{(\delta x_e)} - \frac{k_w(T_P - T_W)}{(\delta x_w)} + \bar{S}\Delta x = 0 \tag{6}$$

where, $\bar{S}$ is the average value of S over the control volume. It is useful to cast the discretization equation (6) into the following form:

$$a_P T_P = (a_E T_E + a_W T_W) + b \tag{7}$$

where,

$$\begin{aligned} a_E &= \frac{k_E}{(\delta x)_E} \\ a_W &= \frac{k_w}{(\delta x)_W} \\ a_P &= a_W + a_E \\ b &= \bar{S}\Delta x \end{aligned} \tag{8}$$

4. TREATMENT OF RADIATION HEAT LOSS TERM

The treatment of radiation heat loss term is considered in equation (4). As shown in equation (1), this term is a function of the dependent variable T itself and it is then desirable to acknowledge this dependence in constructing the discretization equation.

The discretization equation is solved by the technique for linear algebraic equations and only a linear dependence can be accountable.

The average value $\bar{S}$ is expressed as:

$$\bar{S} = S_C + S_P T_P \tag{9}$$

where, S_C stands for the constant part of $\bar{S}$, while S_P is the coefficient of T_P. With the linearized heat loss expression, the discretization equation would still look like Eq. (7), but the two coefficient definitions in equation (8) would change. The new definitions for those two are expressed as:

$$\begin{aligned} a_P &= a_W + a_E - S_P \Delta x \\ b &= S_C \Delta x \end{aligned} \tag{10}$$

The comparison between equations (1) and (4) shows that S is the function of T^4. Thus, the heat loss term is linearized by replacing T^4 by $\left[T^*\right]^4 + 4\left[T^*\right]^3\left[T - T^*\right]$ wherever it appears. Here, the superscript, *, refers to a previous iteration value. This technique of linearization is on the basis of the Taylor series [7].

To solve the set of matrices from equations (7) to (10), TDMA[3] technique [7] is used.

5. RESULTS AND DISCUSSION

In the present work, the temperature distribution on a set of heat pipe for the temperature dependent thermal conductivity of fin material is evaluated. The present results are compared with those of Naumann [4] and Arslanturk [5].

For more precise and also simpler analysis, some dimensionless parameters are defined as follows:

$$\theta = \frac{T}{T_b} \qquad \psi = \frac{2\varepsilon\sigma w^2 T_b^3}{kD} \qquad \xi = \frac{x}{w} \qquad \beta = \lambda T_b \tag{11}$$

where, θ is dimensionless temperature, ψ is a thermo-geometric parameter, ξ is dimensionless length, and β is a dimensionless coefficient for thermal conductivity function. The dimensionless thermo-geometric parameter ψ is a function of some thermal, optical and geometrical properties. In this paper ψ is varied by length (w) and other parameters are constant.

[3] Tri Diagonal-Matrix Algorithm

The present predictions on the dimensionless fin temperature distribution in the axial direction (i.e., fin length, *x*) are depicted by Fig. 3, for constant thermal conductivity $(\beta = 0)$. The fin base temperature and the other variables are assumed as: $T_b = 700K,\ k_b = 257\,W/m.K,\ \varepsilon = 0.85,\ w = 0.04952m$
It can be seen that the temperature decreases as the fin length increases. Also, the present numerical results are directly compared with those available by Naumann [4] and good agreement is shown.

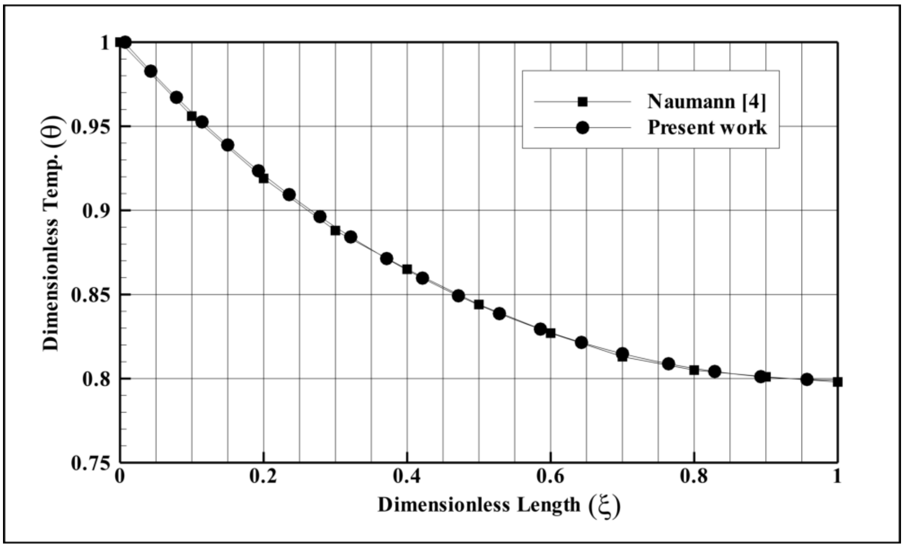

Fig.3 Dimensionless fin temperature distribution for constant thermal conductivity $(\beta = 0)$

Fig. 4 represents the dimensionless temperature distribution at a fixed thermo-geometric parameter, ψ, and for different β in the range 0.0-0.6. It is observed that as β is increased from 0.0 toward 0.6 the temperature difference between the fin base and its tip is decreased and consequently the fin can radiate to outer space with an overally higher temperature. This may lead to an increase in the fin overall efficiency. The values of conductivity of Al at $T = 100K$ and $T = 1000K$ can be taken from the literature [8] as $k = 300\,W/m.K$ and $k = 200\,W/m.K$, respectively. Assuming that the minimum temperature within the fin, i.e., tip temperature, is $100K$, and employing equations (2) and (11) can be found that the thermal conductivity parameter can be found as $w = -0.55$ for the case of variable conductivity. The average conductivity at the same temperature range is $k = 250\,W/m.K$.

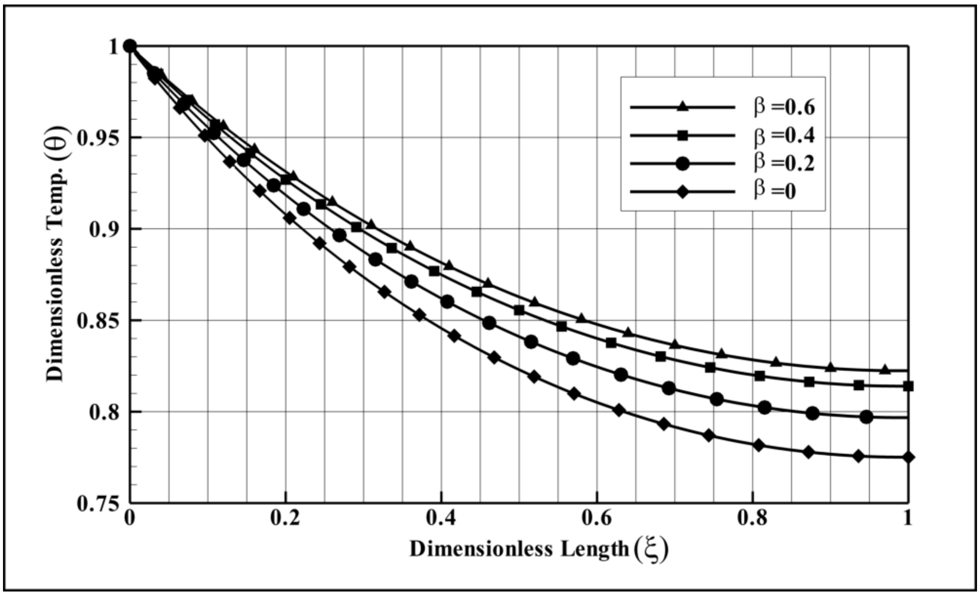

Fig.4 Dimensionless temperature distribution for different β at fixed $\psi = 1$

Fig. 5 demonstrates the distribution of fin temperature for different negative values of β in the range -0.4 to 0.0, at fixed $\psi = 1$. The results for negative values of β are in line with those for positive β in Fig. 4. That is, as β is increased from -0.4 toward 0.0 the temperature difference between the base and tip is decreased.

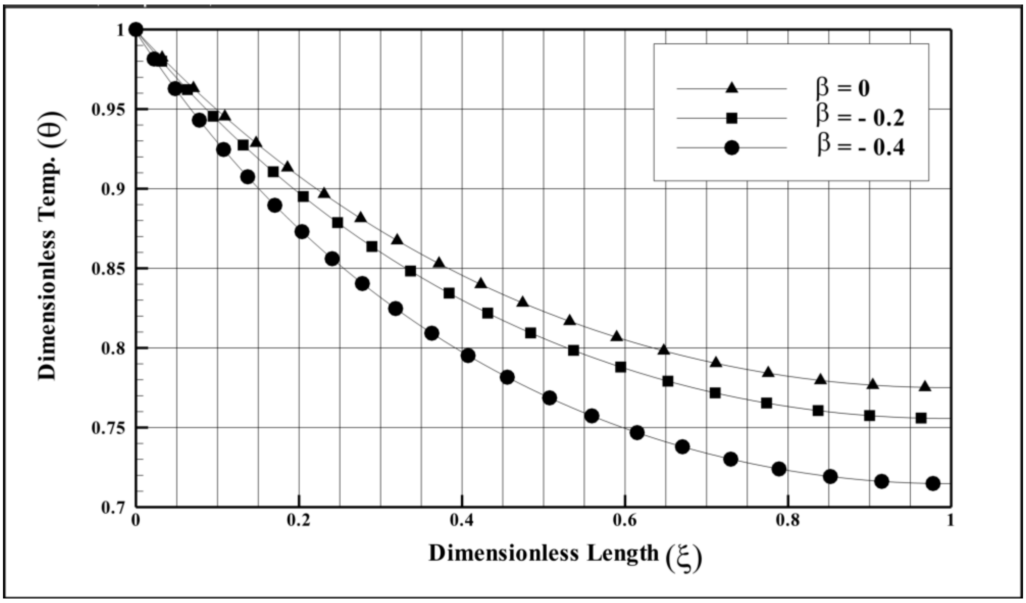

Fig.5 Dimensionless temperature distribution for different negative β at fixed $\psi = 1$

The present dimensionless fin tip temperature predictions (i.e., at x=w) for different β, at constant $\psi = 1$, are depicted in Table 1. They are compared with the two recently available analytical solutions of Arslanturk [5] and Hosseini and Ghanbarpour [6]. The percentage of deviation is less than 5% in all cases. Hence, it can be concluded that the present numerical computations are in good agreement with the works of others.

Table1. Dimensionless tip temperature at $\psi = 1$

Method	$\beta = -0.4$	$\beta = -0.2$	$\beta = 0$	$\beta = 0.2$	$\beta = 0.4$	$\beta = 0.6$
Present work	0.714776	0.755708	0.775102	0.796743	0.813942	0.822373
Reference [5]	0.713201	0.754132	0.779145	0.797712	0.813369	0.82675
Reference [6]	0.712155	0.756802	0.775333	0.797809	0.813236	0.825052

As mentioned before, the thermo-geometric parameter, ψ, is an important parameter and its influence on the fin axial temperature distribution, at fixed $\beta = 1$, is displayed in Fig.6. It is observed that as ψ is increased from 1 to 10, the fin tip temperature and hence overall temperature along the fin length are decreased significantly.

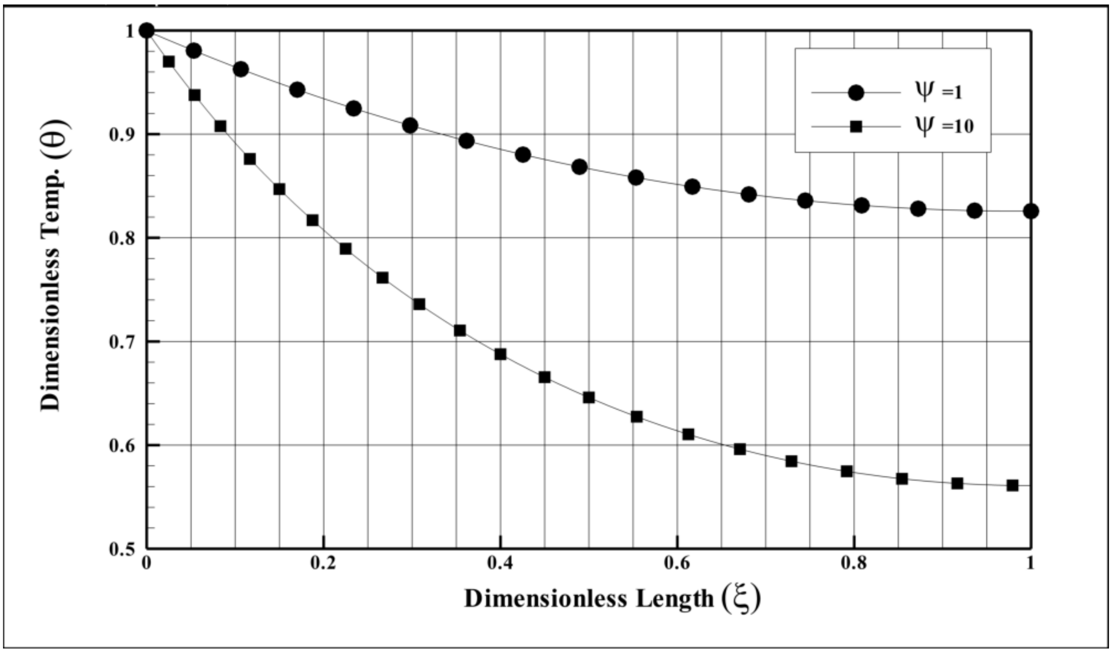

Fig.6 Dimensionless temperature distribution for different ψ , at fixed $\beta = 1$

6. CONCLUSIONS

In the present work, the heat pipe-fin configuration concerning a space radiating fin with the temperature dependent thermal conductivity of the fin material is investigated numerically. The nonlinear governing equation is solved by the finite volume method. Also, the linearization technique is used for the radiation heat loss term. The temperature distribution along a radiating fin is predicted for different geometric and thermal parameters. The main conclusions may be drawn as follows:

- For a constant thermo-geometrical parameter, ψ the fin tip temperature is increased while the dimensionless coefficient for thermal conductivity β is increased in the range -0.4 to 0.6.
- Fin tip temperature decreases if ψ increases from 1 to 10, at constant β.
- Dimensionless tip temperature predictions are compared with two other analytical solutions displaying good agreement with one another.

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