Modelling single facility multi-item batch production planning under regular demand

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1- Abstract

A non linear mathematical programming model is developed for the planning of multi-item common batching cycle production, in a single facility, under conditions of regular demand. The model seeks to maximise the annual net profits through the optimization of the choice of the product mix and observing the relevant capacity constraints, market demands and production policy decisions. An iterative successive approximation procedure is proposed to reduce the model into linear programming iterations that quickly converge towards the solution. On trying the model, it successfully and quickly furnished the required results, proving that the model and the solution procedure efficiently met the set objectives. The model can result in significant improvement in the annual net profit.

Key words: Production planning modeling, multiple item batch production, common batching cycle production.

2- Introduction

Production is one of the main, if not the main, primary functions of any industrial facility. Planning this function depends on the type of facility, the nature of the items involved and the prevailing market conditions. A single facility environment is conceived as an industrial establishment, consisting of an ordered set of interactive interdependent components that can be treated as a single entity. In a single facility distinct inputs are processed in an integrated manner (practically by most of the components of the facility) for the purpose of making distinct outputs. A single facility may make a single item or multiple items that are of similar manufacturing requirements. Strictly speaking, for the efficient exclusive production of a single item in a single facility, a flow line production on the basis of a product layout is the most appropriate. Multiple item production in a single facility environment, by necessity, is planned in batches.

The basic single delivery economic batch quantity model, its variant the continuous delivery model and its extension the quantity discount model are some examples of the treatment of batch production planning (Krajewski and Ritzman 2001). These models, on the bases of annual demands, setting up costs, stock holding costs and sometimes order sizes and quantity discounts, result in the definition of batch quantities, batching cycle times, production period durations up to the maximum inventory levels and depletion periods up to the ends of cycle times. These models do not take into consideration the overall production capacity of the relevant facility and the possibility of optimising the response to market demands through the deliberate choice of the product mix. Additionally, for different items considered separately, different levels of annual demands, setting up costs and stock holding costs result in different batching cycles, production periods and depletion periods. The synchronization of such batching cycles can practically be a formidable task. For these reasons, facilities resort to the adoption of a common batching cycle time where, in each cycle, practically, all items are accommodated.

Viswanathan and Goyal (1997, 2000) dealt with common batch cycling involving shelf lives and back order considerations. Moon et al. (2002) proposed mathematical models for dealing with the economic lot scheduling problem using a common cycle approach and time-varying lot sizes. These models addressed the special problems of imperfect production processes. Sharma (2004) addressed this problem too. In another paper Sharma (2007) analyzed an approach in which the production rates of two items were varied. Giri et al. (2005) proposed an economic production lot size model that can take care of increasing demand, shortages and backlogging. Perhaps the most comprehensive treatment of this type was presented by Sharma (2007[a]). He analyzed the problem involving set up cost, inventory holding cost, shortages, capacity constraints, input item ordering cost and shelf life constraints. All common batch cycling models start with a generalized production cost function that assumes given demand rates and proceed towards an optimal batching cycle time through differentiation or partial differentiation of that function. For a shelf life constraint Sharma (2007[a]) considered three alternative options for dealing with the problem. The treatment involved further partial differentiation and the introduction of some new feasibility constraints. These models, in addition to involving extensive mathematical functions, are sometimes based on over simplified hardly justifiable assumptions. These models also do not take into consideration the optimization of the product mix.

Some more complicated models were also proposed. Miller et al. (2003) presented a generalised mixed integer programming model with polyhedral structure. The model can provide for the relaxation of the various capacitated production planning problems that arise, as substructures, in many industrial applications. In another paper they (2003[a]) described a polynomial algorithm for dealing with the special case of the PI (preceding inventory), namely the PIC, where setup times and demands are constant for all items. For dealing with a multiple period multi-item production with seasonal demand patterns, Ketzenberg et al. (2006) developed a complicated probabilistic multifunction dynamic model. Believing that an optimal solution based on this model, for a real world problem, was impractical, they proposed an equally complicated heuristic procedure to solve it.

In real world industrial production environments, simple yet realistic, models are much more sought after. As such, for common cycle batch scheduling, the development of a simple mathematical programming model that takes into consideration the appropriate choice of the product mix and observes the realistic constraints of production capacity, market demands and production policy, as well as the development of an iterative procedure to solve the model, are going to be the objective of this investigation. To keep the model as simple as practically desired, some of the pertinent issues are proposed to be reasonably treated outside the model. This applies in particular to the treatment of shelf life requirements.

3- Concept and formulation of the model

The targeted model starts with the consideration of the market demands and available capacity for the purpose of the optimization of operational profits. This is achieved through optimizing the choice of the product mix and the common batching cycle time under the prevailing constraints. The objective of the model is the maximization of the annual operational profits, defined as the difference between net revenues and overall costs. Overall production costs are assessed as the sum of all the variable and fixed costs. Net revenues are assessed as functions of the product mix and the sale prices. The prevailing constraints include production capacity, market demands and policy decisions. Possible probabilistic shortages are assumed to be handled, as practically dealt with, by the provision of safety stocks. The possibility of backlogging will not be considered, as it is not the desirable standard policy situation. Input item ordering (dependent demands) will not be involved in the targeted model. This task, as commonly dealt with, can be better optimised separately using materials requirement planning or other relevant techniques.

For the mathematical formulation of the model all designations referring to time will be in years and all designations referring to prices, revenues, costs, profits and other financial parameters can be in the relevant monetary units. All designations referring to output can be in the relevant output measurement units. For the proposed approximation solution procedure of the targeted non linear mathematical programming model, an iterative approach involving (t) as a reference index to indicate the relevant iteration, will be considered.

3.1- Formulation of the objective function

The annual net revenues relevant to iteration (t), namely AR_t, can be evaluated by using the following constraint:

$AR_t - \sum_i p_i X_{t,i} = 0.0$ Where:

i = item general reference index,
$X_{t,i}$ = decided annual output in terms of item (i) according to iteration (t) (This is, in effect, the rate of depletion per annum),
p_i = net market unit price of item (i).

The annual overall production cost relevant to iteration (t), namely AC_t, comprises variable input costs, annualized overall fixed costs M (including those of safety stock holding), variable items set up costs and variable stock holding costs. Considering any iteration (t) and designating:

v_i = variable inputs unit cost of item (i),
M = annualized overall fixed cost of the facility including those of safety stocks (This is obtained from the accounts department),
A_i= item (i) setting up costs,
T_t = common batching cycle time, in years, for all items relevant to iteration (t),
f = stock holding cost factor as a percentage of the market price of the items,
C_i = facility annual output capacity in terms of item (i) when dedicated to the production of item (i) only (This is, in effect, the rate of production per annum),

the annual variable inputs cost can be evaluated as

$\sum_i v_i X_{t,i}$. As T_t is the common batching cycle duration in years, the number of batching cycles per year is $\frac{1}{T_t}$. Accordingly, the annual variable items setting up cost is expressed as $\frac{1}{T_t}\sum_i A_{t,i}$. The assessment of the annual variable stock holding cost requires the assessment of the average stock level over the year. This is, in fact, equal to the average stock level over the batching cycle. As the output of item (i) over the batching cycle is equal to $T_t X_{t,i}$ and considering concurrent production at the rate of C_i and depletion at the rate of $X_{t,i}$, the maximum variable stock level (less the safety stock) of item (i) is $T_t X_{t,i}(1-\frac{X_{t,i}}{C_i})$ (Krajewski and Ritzman 2001). Accordingly, the minimum variable stock level (less the safety stock) is zero, the annual average stock level is $\frac{1}{2}T_t X_{t,i}(1-\frac{X_{t,i}}{C_i})$. As such, the annual variable stock holding cost for item (i) is $\left(f\ p_i\right)\left[\frac{1}{2}T_t X_{t,i}(1-\frac{X_{t,i}}{C_i})\right]$ and the total annual variable stock holding cost for all items is $\frac{fT_t}{2}\sum_i p_i X_{t,i}(1-\frac{X_{t,i}}{C_i})$. This way the annual overall production costs AC_t comprising the elements of variable input costs, annualized overall fixed costs M (including those of safety stock holding), variable items set up costs and variable stock holding costs is assessed as follows:

$AC_t = \sum_i v_i X_{t,i} + M + \frac{1}{T_t}\sum_i A_{t,i} + \frac{fT_t}{2}\sum_i p_i X_{t,i}(1-\frac{X_{t,i}}{C_i})$. When defining E_t as a cost function that includes the annualized overall fixed cost of the facility M (including those of the safety stock), variable item setting up costs and variable stock holding cost, the annual overall production cost AC_t constraint is formulated as:

$AC_t - \sum_i v_i X_{t,i} - E_t = 0.0$, where $E_t = M + \frac{1}{T_t}\sum_i A_{t,i} + \frac{fT_t}{2}\sum_i p_i X_{t,i}(1-\frac{X_{t,i}}{C_i})$.

As the objective function maximizes annual operational profits AP_t, this function together with the relevant definition constraints, are expressed as follows:

$$\text{Max } AP_t = AR_t - AC_t$$

S.T.

$$AR_t - \sum_i p_i X_{t,i} = 0.0$$

$$AC_t - \sum_i v_i X_{t,i} - E_t = 0.0$$

$$E_t - M - \frac{1}{T_t}\sum_i A_{t,i} - \frac{fT_t}{2}\sum_i p_i X_{t,i}(1-\frac{X_{t,i}}{C_i}) = 0$$

3.2- Formulation of the remaining model constraints

In addition to the definition constraints related to the objective function the remaining constraints are formulated as follows:

a- Capacity constraint

The capacity constraint of the facility is formulated using the concept of total equivalent production expressed in terms of a chosen item (i = s). This way the total equivalent production, expressed in terms of the chosen item (s), should not exceed the facility production capacity when completely dedicated to that particular item. Given C_i as the facility annual output capacity in terms of item (i) when dedicated to the production of item (i) only, the unit output of any item (i) is equivalent to $\frac{C_s}{C_i}$ units of the chosen item (s) and the total equivalent production expressed in terms of the chosen item (s) is $\sum_i \frac{C_s}{C_i} X_{t,i}$. The making of the product mix takes place in batches. This implies stoppages during the batch, for the durations of the setting up activities of all the items amounting to $\sum_i t_i$. When considering all the batches over the year, this amount is multiplied by the number of batches per year $\frac{1}{T_t}$. This, for a whole year, amounts to $\frac{1}{T_t}\sum_i t_i$. Accordingly, the net output in terms of the chosen item (s), taking into consideration set up stoppages, is $C_s(1-\frac{1}{T_t}\sum_i t_i)$. This way, the capacity constraint is formulated as:

$$\sum_i \frac{C_s}{C_i} X_{t,i} < C_s(1-\frac{1}{T_t}\sum_i t_i).$$

b- Production related constraints

The production of any item should not exceed market demand. Additionally, in many cases, for various reasons, a management decision may fix a minimum output level for some items. Designating EXi as the externally specified minimum annual output in terms of item (i) and designating D_i as the market annual demand for item (i), these restrictions are modeled as follows:

$X_{t,i} < D_i$, A constraint for each (i), and

$X_{t,i} > EX_i$, A constraint for each relevant (i).

c- Management information requirements definition constraints

The assessments of the batch quantity of the each item $Q_{t,I}$, duration of the production period $PT_{t,i}$, sum of the production periods of all items during the common batching cycle time PT_t and depletion period for each item $DT_{t,i}$, as well as the capacity utilization of the facility CU_t, are prime management information requirement. The provisions for the assessment of these requirements are addressed as follows:

$Q_{t,i} - T_t X_{t,i} = 0.0$, A constraint for each (i)

$$PT_{t,i} - \frac{Q_{t,i}}{C_i} = 0.0 \quad \text{, A constraint for each (i)}$$
$$DT_{t,i} + PT_{t,i} = T_t \quad \text{, A constraint for each (i)}$$
$$PT_t - \sum_i PT_{t,i} = 0.0$$
$$CU_t - \frac{PT_t}{T_t} = 0.0$$

This way the required model is put together as follows:

$$\text{Max } AP_t = AR_t - AC_t$$
$$AR_t - \sum_i p_i X_{t,i} = 0.0$$
$$AC_t - \sum_i v_i X_{t,i} - E_t = 0.0$$
$$E_t - M - \frac{1}{T_t}\sum_i A_i - \frac{fT_t}{2}\sum_i p_i X_{t,i}(1 - \frac{X_{t,i}}{C_i}) = 0.0$$
$$\sum_i \frac{C_s}{C_i} X_{t,i} < C_s(1 - \frac{1}{T_t}\sum_i t_i)$$
$$X_{t,i} < D_i \quad \text{, A constraint for each (i)}$$
$$X_{t,i} > EX_i \quad \text{, A constraint for each relevant (i)}$$
$$Q_{t,i} - T_t X_{t,i} = 0.0 \quad \text{, A constraint for each (i)}$$
$$PT_{t,i} - \frac{Q_{t,i}}{C_i} = 0.0 \quad \text{, A constraint for each (i)}$$
$$DT_{t,i} + PT_{t,i} = T_t \quad \text{, A constraint for each (i)}$$
$$PT_t - \sum_i PT_{t,i} = 0.0$$
$$CU_t - \frac{PT_t}{T_t} = 0.0$$

4- Model's solution procedure

The model above is a non linear mathematical program involving terms of third degree. Substituting for AR_t and AC_t in the objective function's expression, the objective function becomes:

$$\text{Max } AP_t = \sum_i p_i X_{t,i} - \sum_i v_i X_{t,i} - E_t$$
$$= \sum_i \left[X_{t,i}\left(p_i - v_i\right)\right] - E_i .$$

As a positive difference between the net unit prices and variable input unit costs of the items is normally a prerequisite condition for any production activity, the maximization of the annual operational profits AP_t depends on the maximization of capacity utilization and the minimization of the cost function E_t. Considering the capacity utilization constraint $\sum_i \frac{C_s}{C_i} X_{t,i} < C_s(1 - \frac{1}{T_t}\sum_i t_i)$, and as the value of $\sum_i t_i$ is normally small in comparison with T_t, capacity is only slightly affected by changes in T_t. This leaves the maximization of the annual operational profits Ap_t mainly dependent on the minimization of the cost function E_t.

The cost function E_t relevant to any iteration (t), was expressed as:

$E_t = M + \frac{1}{T_t}\sum_i A_i + \frac{fT_t}{2}\sum_i p_i X_{t,i}(1 - \frac{X_{t,i}}{C_i})$. By simple differentiation with respect to T_t, for any given set of values of ($X_{t,i}$'s), (implying the treatment of $X_{t,i}$'s as fixed quantities) it can be proved that the minimum value of E_t, namely E_t^*, is associated with an optimal value of T_t, namely T_t^*, evaluated as follows:

$$T_t^* = \sqrt{2\sum_i A_i /(f\sum_i p_i X_{t,i}(1 - X_{t,i}/C_i))} \qquad \text{---------- (1).}$$

Substituting the value (T_t^*) in the expression of E_t gives:

$$E_t^* = M + \sqrt{2f\sum_i A_i \sum_i (p_i X_{t,i}(1 - X_{t,i}/C_i))} \qquad \text{----------(2)}$$

Starting with an external estimate of ($X_{t,i}$'s), for any iteration (t), it is possible to externally estimate T_t^* and E_t^* using equations (1) and (2), This enables the treatment of T_t and E_t as constants and consequently enables the elimination of the constraint:

$$E_t - M - \frac{1}{T_t}\sum_i A_i - \frac{fT_t}{2}\sum_i p_i X_{t,i}(1 - \frac{X_{t,i}}{C_i}) = 0.0 .$$

This results in the reduction of the non linear model to the following iterative linear programming general formulation:

Max $AP_t = AR_t - AC_t$

$AR_t - \sum_i p_i X_{t,i} = 0.0$

$AC_t - \sum_i v_i X_{t,i} = E_t$

$\sum_i \frac{C_s}{C_i} X_{t,i} < C_s(1 - \frac{1}{T_t}\sum_i t_i)$

$X_{t,i} < D_i$, A constraint for each (i)

$X_{t,i} > EX_i$, A constraint for each relevant (i)

$Q_{t,i} - T_t X_{t,i} = 0.0$, A constraint for each (i)

$PT_{t,i} - \frac{Q_{t,i}}{C_i} = 0.0$, A constraint for each (i)

$DT_{t,i} + PT_{t,i} = T_t$, A constraint for each (i)

$PT_t - \sum_i PT_{t,i} = 0.0$

$CU_t - \frac{PT_t}{T_t} = 0.0$

For the model's iterative solution procedure, initially, the values of T_1^* and E_1^* are externally computed using the market demands D_i's for the values of the assumed output of the products (i's). Based on this external estimates of T_t^* and E_t^* the solution of the iterative formulation, for any iteration (t), results in a definition of the corresponding values of ($X_{t,i}$'s). On the basis of these values, using equations (1) and

(2), better external estimates of the corresponding values of T^* and E^*, respectively termed T^*_{t+1} and $E^*_{t+1,}$ are made. Relevant to any iteration (t), the iterative reduction of the differences between the starting value E_t^* and the resulting (better estimated) value E^*_{t+1} will be the basis of the solution approximation. As the functions $\frac{1}{T_t}\sum_i A_i$ and $\frac{fT_t}{2}\sum_i p_i X_{t,i}(1-\frac{X_{t,i}}{C_i})$ act in opposite directions and as the rates of annual production are only slightly affected (as shall be seen in the application example) by changes in the value of T_t, the external estimate of T_t^* and E_t^* tends to make the model's iterations quickly converge toward a near optimal solution. Relevant to any iteration (t), the decision for another iteration (t+1) is only made if the approximation control index $AI_t = (E_t - E_{t+1})/E_t$ is greater than LAI. The parameter LAI is the externally set maximum acceptable value of the approximation control index. In the case of a decision for another iteration (t+1), the values of T^*_{t+1} and E^*_{t+1}, assessed on the basis of the previous iteration (t), are used as external estimates.

This approximation procedure, as charted in Figure 1, is elaborated as follows:
- For the assessment of T^*_1 and E^*_1 for the initial iteration (t=1), all market demands are assumed to be met; this implies assuming the output of items (i's) equal to the annual demands (D_i's) of the items. Using equations 1 and 2, the value of T^*_1 and E^*_1 are externally computed. The value of E^*_1, estimated this way, is the highest that can be expected. Substituting the values of T^*_1 and E^*_1 and solving this iteration linear program result in the definition of ($X_{1,i}$'s). On the basis of these values of ($X_{1,i}$'s), estimates of T^*_2 and E^*_2 are made. As this is the initial iteration, where the assumed values of T^*_t and E^*_t are obviously far from being true, the value of AI_1 is inevitably high, deeming another iteration necessary.
- The next iteration (t=2) is executed substituting the already obtained values of T^*_2 and E^*_2. The solution of the resulting linear program leads to the definition of revised values of ($X_{t,i}$'s). These values, in turn, are used for the evaluation of the of T^*_{t+1} and E^*_{t+1}, which are used for the next iteration, if required. The solution also enables the evaluation of (AI_t), that can lead to another iteration when $AI_t > LAI$.
- Another iteration (t=t+1) is executed as long as $AI_t > LAI$. This is achieved by substituting the relevant values of T^*_{t+1} and E^*_{t+1} already computed on the basis of the previous iteration. This procedure is typically repeated, using the iterative formulation of the model and acting as outlined in connection with iteration (t), until a satisfactory solution is achieved. When a satisfactory solution is achieved the values of ($X_{t,i}$'s), (T_t), ($Q_{t,i}$'s), ($PT_{t,i}$'s) and ($DT_{t,i}$'s) are adopted for the production plan.

5- Trial of the model

The trial of the model is best illustrated using an application example. To solve the iterative linear programmes, Hyper Lingo 9 PC (2004), was used. The externally assessed input data, used for the illustration example, were based on an economic and engineering analysis of a real past year situation, where a facility making standard wooden furniture was engaged on the production of three items namely: 1, a master bedroom set; 2, reception room set and 3, dining room set. The details of the input data are as summarised in table 1. Assuming a limiting approximation control index value of 0.0005 and assuming that the annual output of the items (i's) are equal to the market demands (D_i's), the values of T^*_1 and E^*_1 were assessed for the initial iteration (t=1) using equations 1 and 2. These values and other input parameters and coefficients from table 1 were fed into the model resulting, amongst other things, in the improved definition of iteration (1) annual production rates of the three items $X_{1,1}$, $X_{1,2}$ and $X_{1,3}$. Accordingly, on the basis of the improved values of $X_{1,1}$, $X_{1,2}$ and $X_{1,3}$, the values of T^*_2 and E^*_2 were computed for the next iteration. Using the values of E^*_1 and E^*_2, the value of AI_1 was computed as 0.0058. As this value is greater than 0.0005, another iteration was required. Using the improved values of T^*_2 and E^*_2 and the input data of table 1, iteration 2 was executed resulting in the definition of the iteration production rates $X_{2,1}$, $X_{2,2}$ and $X_{2,3}$ of the three items: Substituting these values in equations 1 and 2, the values of T^*_3 and E^*_3 were assessed. Using the values of E^*_2 and E^*_3 the value of AI_2 was computed as 0.00008. This value obviates the need for another iteration thus making the solution of iteration 2 as the accepted solution. A summary of the solution results is presented under iterations 1 and 2 of table 2. As such the common cycle batch production should be scheduled as shown in table 3

As evident, the production periods add up to 0.1 year. The remaining time of (0.0067) years of the batching cycle is the time of the setting up activities of the three items.

6- Discussion, conclusions and recommendations

From the course of this investigation, it can be seen that the model developed and the proposed iterative successive approximation solution procedure were capable of meeting the set objectives. The model managed to optimize the choice of the product mix and batch scheduling, observing the constraints of production capacity, market demands and production policy decisions. Due to the nature of mathematical programming, any production policy requirements, other than minimum output volumes, can be easily accommodated. Any particular constraints relating to the nature of the relevant facility can also be easily accommodated. As demonstrated by the application example the model converged on the solution in a few iterations. When lower values of LAI are set, more accuracy, at the cost of more iteration, is always possible.

The running of the model is the responsibility of the capacity and production planning and control function. With reasonable training, the schedulers of the establishment, where the model was tried, were able to successfully use the model. The whole solution of the application example, including the final preparation of the production planning recommendations, did not take more than one hour. This period, production planners can easily afford. The estimate of the would be improvement in net annual profits, over the estimate of the accounts profits made by the case establishment, on the basis of using the isolated continuous delivery economic batch quantity model for each of the items on its own, during the relevant year, was around 6.3 %. This is explained by taking into account the capacity constraint, the proper choice of the production programme and the cycle scheduling. Likewise benefits can be obtained elsewhere.

The input data required for the application of the model, as listed in Table1, is the standard types of data that can be made available by the various management functions. The annual demand (D_i) as well as the unit market price (p_i) of any product can be made available by the marketing function. The Exclusive annual capacity (C_i) and the product variable set up time (t_i) can be assessed by the production planning function. The variable product set up costs (A_i), the stock holding cost factor (f), the product variable input unit cost (v_i) and the annualized overall fixed costs (M) can all be made available by the accounting function. Finally the information on any externally fixed annual production (EX_i) can be obtained from the relevant decision maker.

The model's solution information, as shown in table 2, can be used for the various management functions. The capacity utilization, (CU_t), the iteration common batching cycle time (T_t), the annual production rate ($X_{t,i}$), the iteration batch quantity ($Q_{t,i}$), the iteration production time ($PT_{t,i}$) and the iteration depletion time ($DT_{t,i}$) of any product can be used for the production planning and/or control functions. Additionally all the financial information can be used for the cost control function.

Taking into account any particular aspects of the relevant facility and production items, this kind of model, is recommended for single facility multiple item production planning. The model's integration into the computerized framework of any relevant facility can be a subject for another investigation.

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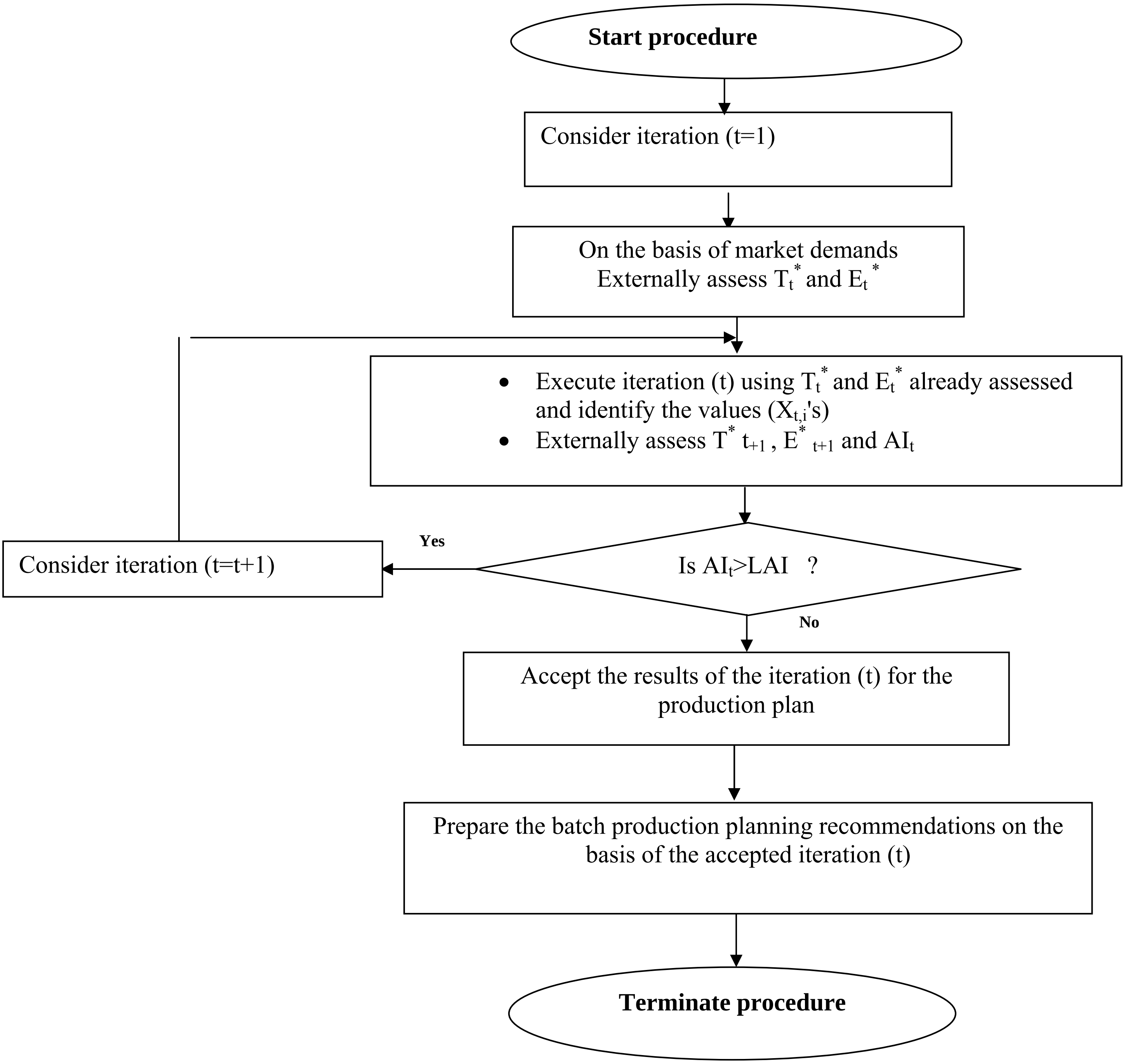

Figure 1: Iterative successive approximation solution procedure

Table 1: Externally assessed model's input data details

Input data details	Item no. (i)			Remarks
	1	2	3	
Annual demand (D_i) in number of units	1500	1100	700	$\sum A_i$=1755 and $\sum t_i$=0.007 for a no shelf life constraint.
Exclusive annual capacity (C_i) in number of units	3000	2500	2500	
Externally fixed annual production (EX_i) in number of units	-	-	300	
Variable product set up costs (A_i) in dollars	670	680	405	
Net market unit price (p_i) in dollars	1000	1300	800	
Stock holding cost factor (f)	0.18	0.18	0.18	
Product variable input unit cost (v_i) in dollars	600	800	350	
Product variable set up time (t_i) in years	0.003	0.002	0.002	
Annualized overall fixed costs (M) in dollars	350000			

Table 2: Model's iterative solutions details

Iterative solution details	Iteration no. (t)	
	1	2
Iteration common batching cycle time (T_t)	0.1	0.1067
Iteration cost function (E_t)	385136	382882
Iteration net profits (AP_t)	743864	751318
Iteration net revenues (AR_t)	2780000	2793000
Iteration net costs (AC_t)	2036136	2041682
Annual production rate of product 1, ($X_{t,1}$)	1110	1123
Iteration batch quantity of product 1, ($Q_{t,1}$)	111	120
Iteration production time of product 1, ($PT_{t,1}$)	0.037	0.04
Iteration depletion time of product 1, ($DT_{t,1}$)	0.063	0.066
Annual production rate of product 2, ($X_{t,2}$)	1100	1100
Iteration batch quantity of product 2, ($Q_{t,2}$)	110	117
Iteration production time of product 2, ($PT_{t,2}$)	0.044	0.047
Iteration depletion time of product 2, ($DT_{t,2}$)	0.056	0.06
Annual production rate of product 3, ($X_{t,3}$)	300	300
Iteration batch quantity of product 3, ($Q_{t,3}$)	30	32
Iteration production time of product 3, ($PT_{t,3}$)	0.012	0.013
Iteration depletion time of product 3, ($DT_{t,3}$)	0.088	0.094
Iteration sum of the common batching cycle production times, (PT_t)	0.093	0.1
Iteration capacity utilization, (CU_t)	0.93	0.93
Next iteration common batching cycle time (T_{t+1})	0.1067	0.1066
Next iteration cost function (E_{t+1})	382882	382914
Iteration approximation control index (AI_t)	0.0058	0.00008
Decision for another iteration	Yes	No

Table 3: Summary of the common cycle batch production schedule

Production item (i)	1	2	3
Production period ($PT_{t,i}$) in years	0.04	0.047	0.013
Item batch quantity ($Q_{t,i}$) in number of units	120	117	32

Captions for Figures and Tables

Figure 1: Iterative successive approximation solution procedure
Table 1: Model's externally assessed input data details
Table 2: Model's iterative solutions details
Table 3: Summary of the common cycle batch production schedule