Stochastic reliability of unreinforced masonry walls subjected to blast

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***Abstract*— Unreinforced bricks masonry (UBM) is widely used for construction of walls. However, due to their weight and low blast resistance, they can present a significant hazard to occupants when subjected to blast loads. There are considerable uncertainties associated with material properties, threat scenarios, as well as expected damage. In this work, a stochastic simulation is conducted to evaluate the reliability of UBM wall subjected to blast load, thus accounting for these uncertainties. Nonlinear dynamic Finite Element Modeling (FEM) is used to simulate the brick and mortar wall. Sensitivity to input parameters is tested so as to select the major factors. The uncertainties of the major factors are included in the model simulation. In order to reduce computational cost, sampling is performed using LHC technique. The stochastic reliability analysis proved effective for studying the damage risks for UBM walls subjected to blast loadings, where results in the form of Probability Density Function (PDF), Cumulative Distribution Function (CDF), and survival function are obtained. It is shown that the occurrence of total wall collapse is a low possibility.**

Keywords- Stochastic reliability simulation, Dynamic finite element, Unreinforced brick masonry, Explosive blast loading, Latin hypercube sampling.

I. INTRODUCTION

Masonry construction; one of the oldest building techniques, is still widely used today. Since the Oklahoma city bombing in 1995, the awareness of the devastating damage to infrastructure due to explosions has been heightened. The most likely vulnerable infrastructures are old, brittle masonry buildings, especially unreinforced brick masonry (UBM) walls. Blasts acting in the out-of-plane direction could pose the highest risk. UBM walls could present a significant hazard to building occupants in case of a blast event. Therefore, it is of interest to understand the behavior of UBM walls under blast loading. This work is a continuation of the conference paper presented in [1]. An interesting fact about brick and mortar is that they have very different tensile and compressive strengths. Moreover, their strength is sensitive to pressure and strain rate. Hao and Tarasov [2] conducted dynamic uniaxial compressive tests to study the strain rate effect of brick and mortar materials. They found that the compressive strength of brick and mortar increases significantly with the strain rate. Stewart and Lawrence [3] developed a method to calculate the structural reliability of typical masonry walls subject to vertical bending. They found that structural reliabilities are very sensitive to wall width, workmanship, and discreteness of masonry unit thickness. As reported by Hao and Tarasov [2]; a reliable prediction of UBM structure response to blast loads requires an accurate material model. Such model should reflect the characteristics of brick and mortar behavior at high strain rates.

An experiment in which 6 full-sized clay UBM walls were constructed by 4 masons was performed by Heffler et al. [4]. The flexural bond strength of each unit in each wall was obtained. The spatial correlation of unit bond strengths indicated that they are statistically independent [4]. Eamon [5] developed a procedure for reliability assessment of masonry walls subjected to blast load. The method is based on analytical reliability methods and nonlinear Finite Element Model (FEM). They found that the main random variable that affects wall resistance are mortar joint strength and contact surface friction. Weidlinger [6] analyzed a typical reinforced concrete façade cladding panel. It was subjected to blast loads in the form of vapor cloud explosions. They generated pressure-impulse iso-damage curves for the structural component. They found that hand calculations, finite element, and single degree-of-freedom analyses results were all consistent.

A practical theory of explosives blast waves and blast loading of structures was presented by Neff [7]. The theory was used to develop a simplified visual model of explosions for use in computer graphics. El-Domiaty et al. [8] studied the effect of retrofitting un-reinforced masonry using Fiber-Reinforced Polymer composites on the surfaces of the walls. This resulted in increasing the resistance to blast loads, especially the high flexural stresses resistance. Makovička [9] discussed the manner of failure of masonry. They investigated both the origin of the first crack in the structure and the collapse of the whole structure based on tensile stress and deflection. FEM is used to predict the damage of UBM walls subjected to blast loading based on the explicit FEM software LS-DYNA®. The prediction of damage was based on the deflection response of the structure, Wei and Stewart [10]. In this work, the reliability of UBM walls is studied using Monte Carlo Simulation (MCS) in conjunction with Finite Element

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Modeling (FEM). The outer iterations are performed using MCS method, while the inner computations are performed using FEM method. This approach results in qualitative and quantitative estimation of the reliability of UBM subjected to blast loading. The stochastic simulation has considered all factors affecting reliability. However, by means of sensitivity analysis, these factors are divided into 2 groups; Major factors and minor factors. The Probability Density Functions (PDFs) of the major factors are used in the simulation. But since the minor factors have less impact, and therefore their variance can be negligible, the simulation was limited to their mean values without considering their stochastic characteristics.

II. RELIABILITY ESTIMATION METHOD

The reliability prediction process used in this work is shown in Figure 1 [11]. The procedure consists of 3 main computational steps; the first step is to obtain the PDFs of input variables, which are considered random variables. Random input data points are generated using the corresponding PDFs distributions. The sample generation process is conducted using MCS, or more precisely Latin Hypercube (LHC) Sampling. Input parameters such as material strength and wall thickness are treated as random variables in the sampling process. For each iteration; a transient dynamic structural FEM analysis is conducted, where a dynamic plastic damage model is used for brick and mortar. As a result, a deflection response is obtained for each iteration. Due to input parameters uncertainties, the calculated deflection will be in the form of a PDF. Based on this curve, the Cumulative Distribution Function (CDF) of Deflection is drawn. The reliability of UBM wall can be directly read from the Deflection CDF. As it represents the probability that Deflection does not exceed the damage value, and that is the very definition of reliability in this case.

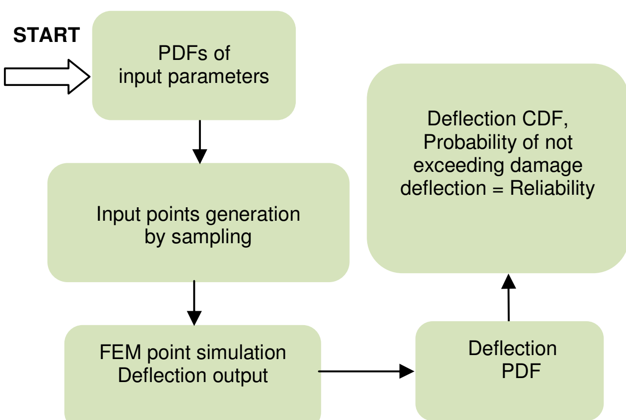

Figure 1. Reliability estimation method

The sampling process is based on LHC sampling technique. This method is used to enhance the sampling process and reduce computational effort. The damage criterion used in this analysis is wall collapse caused when wall deflection exceeds wall thickness [10]. The structural reliability (R_s) is the probability that deflection does not exceed wall thickness as shown in Figure. 2. In mathematical form, the reliability is expressed as:

$$R_s = 1 - P_f = 1 - P\,(\Delta > t_w) \quad (1)$$

Where

P_f : Probability of failure

P : Probability

Δ : Deflection

t_w : Wall thickness.

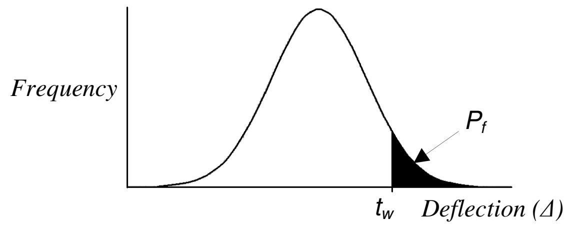

Figure 2. Computation of failure probability

2.1. Monte Carlo Simulation (MCS)

MCS technique is used to estimate the reliability of the UBM wall model. It consists of the following steps [12]:

a- Define the input random variables

b- Quantify the PDFs of the input variables based on their probabilistic characteristics.

c- Generate the values of the random variables.

d- Numerical experimentation for each set of realizations of all the random variables.

e- Extract probabilistic information from N such realizations.

The factors affecting the efficiency of MCS include sampling technique, number of simulations, distributions characteristics, and responses configuration. MCS is a powerful technique, however, it is computationally expensive due to the large number of required sample points, especially using direct sampling. Therefore, an improved version of MCS is employed in this work. This version is called LHC sampling (Figure. 3). LHC sampling is performed by first dividing the domain of each input variable into n strata of equal marginal probability $1/n$, and sample once from each stratum. A LHC design with n runs and s input variables, is an $n \times s$ matrix, in which each column is a random permutation of $\{1,2,\ldots,n\}$ [13]. In other words, LHC technique generates sample points by dividing each random variable distribution into a number of intervals of equal probability. Samples are randomly combined from these intervals for each random variable. As a result, the number of iterations required to estimate the statistical characteristics of the model behavior is reduced. [14].

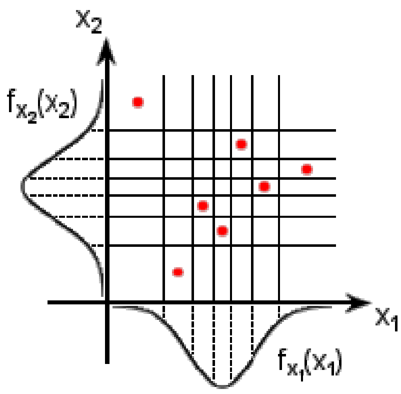

Figure 3. LHC sampling technique [14]

The reliability of the UBM wall is based on the extent of damage, where the damage depends on the amount of transient deflection response. The factors that affect deflection are listed in Tables 1 and 2. A sensitivity analysis is conducted to determine the major factors that should be included in the reliability simulation, where they are considered as random variables. On the other hand, the variation of the minor factors has no significant effect on deflection, therefore, only their mean values are considered in the simulation. The distributions of the major factors are used to generate input values for the stochastic FEM. The number of generated random data points is directly proportional to the number of input variables.

The sensitivity analysis indicated that the major factors affecting wall deflection and hence reliability are; wall thickness, blast pressure, bond strength, as well as Brick and Mortar strengths. It is clear that blast pressure varies according to charge weight and stand-off distance. In addition, the strengths of the bond, brick, and mortar are combined in one weighted value in order to simplify the model and make the number of the FEM iterations manageable. Consequently, the number of variables will be 3; namely wall thickness, blast pressure, and weighted strength. Next, 3 points are generated for each of these random variables using LHC sampling. The permutations of these points are used to run FEM simulations, and the result is 3^3 = 27 random results (based on 27 random FEM iterations). These random results are used to build the PDF of deflection. Since the damage state is related to peak deflection [10], the probability of failure corresponds to the probability of exceeding the associated damaging deflection. As the reliability analysis depends on the defined failure criteria, it is important to select the proper failure criteria at the onset of the simulation.

The most widely used criteria for design of blast resistant structures is the design manual TM5-1300 [15]. Doherty et al. [16] found that walls would not collapse until the mid-height deflection was equal to wall thickness. The US Army Corps of Engineers also recommends that deflection exceeding wall thickness be used as a failure criterion [17]. Zapata and Weggel [18] reported the blast test results on a 2-story unreinforced masonry structure. The field observations indicated that if the extreme deflection is greater than wall thickness, the infill walls will collapse. Moreover, Zapata and Weggel [18] observed that though some walls have not totally collapsed because they were restrained by the friction interlock between brick wall regions, they appeared unstable and probably would have collapsed under the action of a small additional lateral load. In this work, the criterion for structural collapse is when maximum deflection exceeds wall thickness [10].

TABLE I. BRICK AND MORTAR PARAMETERS [10]

Part	Property	Mean	Coeff. of Variation	Distribution
Brick (B30)	$\alpha_t = \alpha_c$	1.0		Constant
	V	0.15		Constant
	σ_{st0} (MPa)	$\sigma_{sc0} * k_{st}$		Constant
	σ_{sc0} (MPa)	30.0	0.1	Normal
	σ_{sttt0} (MPa)	$\sigma_{st0} * k_{sttt}$	0.1	Normal
	ε_{st0}	0.048 * ε_{sc0}		Constant
	ε_{sc0}	0.00212 + 7.5E-5 * σ_{sc0}		Constant
	k_{st}	0.0625	0.2	Normal
	k_{sttt}	0.025		Constant
Mortar (M10)	$\alpha_t = \alpha_c$	1.0		Constant
	V	0.2		Constant
	σ_{st0} (MPa)	$\sigma_{sc0} * k_{st}$		Constant
	σ_{sc0} (MPa)	10.0	0.15	Normal
	σ_{sttt0} (MPa)	$\sigma_{sc0} * k_{sttt}$		Constant
	ε_{st0}	0.166 * ε_{sc0}		Constant
	ε_{sc0}	0.00143 + 0.00016 * σ_{sc0}		Constant
	k_{st}	0.167	0.4	Normal
	k_{sttt}	0.0025		Constant

2.2. Stochastic FEM Simulation

The combination of nonlinear FEM and MCS requires high computational resources. In order to reduce the problem to manageable proportions, LHC sampling is used for MCS. However, the required computational resources are still high. Therefore, a simplified FEM model is used to conduct the FEM simulation. The original model is shown in Figure. 4 and the simplified model is shown in Figure. 6. A transient FEM simulation is set-up to determine the wall response to dynamic pressure impact. The structural analysis aims to calculate the maximum deflection, which is used to predict the extent of damage due to each loading scenario [11]. The random variables distributions selected in section 2.1 are used in the FEM simulation, as well as the means of the parameters shown in Tables 1 and 2.

TABLE II. CHARACTERISTICS OF INPUT PARAMETERS [10]

Property	Mean	Coeff. of Variation	Distribution
Bond Strength (MPa) [4]	0.51	0.8	Weibull
Charge Mass (W)	100 kg		Constant
Stand-off Distance (R)	20 m		Constant
Scaled R (Z) ($m/kg^{1/3}$)	4.3		Constant
Wall Thickness (t)	355 mm	0.03	Normal
Blast Pressure (P)	4.3 MPa	0.002	Normal

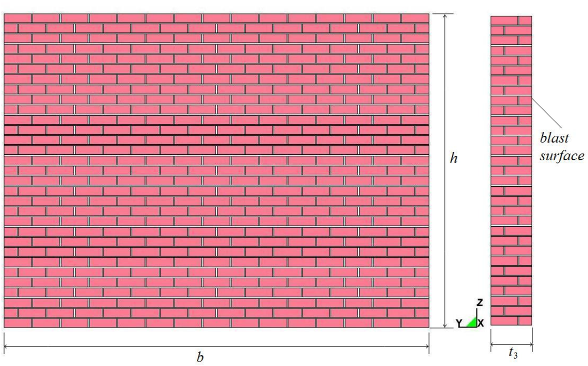

Figure 4. UBM wall model

III. BRICK AND MORTAR MODEL [10]

The 4 sides of the model in Figure. 4 are fixed [10]. A piece-wise Drucker-Prager strength criterion is used to characterize the behavior of brick and mortar. Under high compression, damage of a quasi-brittle material might occur owing to compressive crushing and tensile splitting. A damage scalar consisting of 2 parts is defined based on Mazars theory [19]. The behavior in uniaxial tension and compression is assumed to be linear elastic until the threshold strain is reached. Dynamic uniaxial compressive tests were conducted by Hao and Tarasov [2] to study the strain rate effect on brick and mortar material properties. Formulations of the Dynamic Increase Factor (DIF) for brick material properties were fitted against experimental data for different strain rates [2].

Because of the lack of data on strain rate effects in tension for brick and mortar, it is assumed that the DIF of tensile strength is the same as in compression. It should be noted that it has been found that the strain rate effect on geomaterials such as concrete is more significant on tensile strength than on compressive strength [20], i.e., the DIF for tensile strength at the same strain rate is usually larger than that for compressive strength. The above assumption will most likely under-predict the strain rate effect on brick and mortar tensile strength. Hence, the structural capacity of masonry walls may also be under-predicted. The static uniaxial tensile, uniaxial compressive strength and Poisson's ratio are determined from test data. The triaxial tensile strength is assumed to be about one-third of the uniaxial tensile strength because there is no triaxial tensile test data available [21].

The size of the clay bricks used in the tests was 230 × 115 × 75 mm and a wall was constructed according to Tables 1 and 2. The wall has clear size of $b \times h$ of 3.0 × 3.0 m. 3-dimensional solid elements are used. In order to minimize the effect of mesh size on the numerical results, 3 different meshes are used. As the concern of this study is investigating the global response, convergence of the maximum deflection of the wall is used to optimize mesh size. There are a total of 19,000 elements in the numerical model of the wall. It was found that further refinements in mesh density did not significantly improve global response.

3.1. Brick and Mortar Strength

The strength of brick and mortar is calculated using data in Table 1. Weighted averages of strength values are used in order to reduce the processing time. The weight of brick and mortar is 4 and 1, respectively. Normal distribution is represented as: Norm (Mean, Standard Deviation). The strength properties are calculated as follows:

Maximum tensile strength:

$$\sigma_{st0} = \frac{4 \times norm(30,0.1) \times norm(0.0625,0.2) + norm(0.167,0.4) \times norm(10,0.15)}{5} \tag{2}$$

Maximum compressive strength:

$$\sigma_{sc0} = \frac{4 \times norm(30,0.1) + norm(10,0.15)}{5} \tag{3}$$

Maximum tensile strain:

$$\varepsilon_{st0} = \frac{4 \times 0.048 \times \left(0.00212 + 7.5 \times 10^{-5} \times norm(30,0.1)\right) + 0.166 \times \left(0.00143 + 0.00016 \times norm(10,0.15)\right)}{5} \tag{4}$$

Maximum compressive strain:

$$\varepsilon_{sc0} = \frac{4 \times \left(0.00212 - 7.5 \times 10^{-5} \times norm(30,0.1)\right) + 0.166 \times \left(0.00143 + 0.00016 \times norm(10,0.15)\right)}{5} \tag{5}$$

These equations were executed 200 times using MATLAB and the results were used in the FEM simulation.

IV. BLAST PRESSURE

Peak values of the pressure were considered. The peak value of pressure depends on the radius of the blast sphere, where R is defined as the distance from the center of the blast (meters), and W is the equivalent weight (kilogram) of TNT given in [22]. The scaled R (Z) ($m/kg^{1/3}$) is defined as:

$$Z = \frac{R}{W^{\frac{1}{3}}} \tag{6}$$

The applied pressure is defined as:

$$P[bar] = \begin{cases} \frac{14.072}{Z} + \frac{5.54}{Z^2} - \frac{0.357}{Z^3} + \frac{0.00625}{Z^4}, & 0.05 \le Z < 0.3 \\ \frac{6.194}{Z} - \frac{0.326}{Z^2} - \frac{2.132}{Z^3}, & 0.3 \le Z < 1 \\ \frac{0.662}{Z} + \frac{4.05}{Z^2} + \frac{3.288}{Z^3}, & 1 \le Z < 10 \end{cases} \tag{7}$$

Taking Z = 4.3 [1], and assuming a deviation of 10% ➔ $P = \frac{0.662}{Z} + \frac{4.05}{Z^2} + \frac{3.288}{Z^3}$, after solving for P using 2000 MATLAB iterations, P is obtained as P=norm(4.3,0.002) MPa where 4.3 is the mean and 0.002 is the Coefficient of Variation. The normal distribution formula is:

$$F(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} \tag{8}$$

Where;

$\boldsymbol{x}$: Random variable

$\boldsymbol{\sigma}$: Standard deviation of x

$\boldsymbol{\mu}$: Mean of x

The calculated value for the blast pressure (P) is listed in table 2. Using this value in conjunction with (8) results in the PDF shown in Figure. 5.

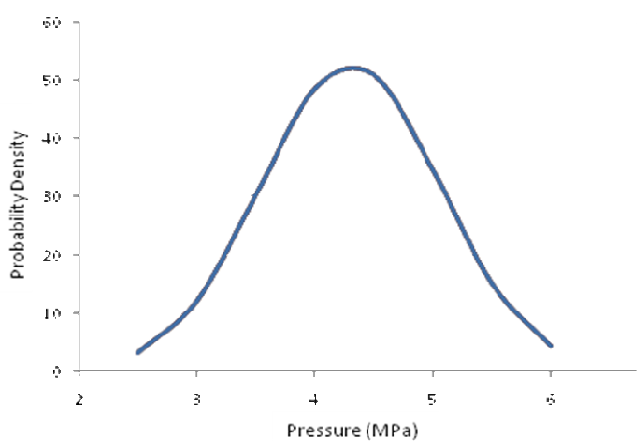

Figure 5. PDF of blast pressure

V. SIMULATION RESULTS

5.1. Inner Loop using Dynamic FEM

The selected input PDFs are used to generate input random variables for the FEM simulation. Sample data points are generated by LHC technique. The maximum deflection is determined for each iteration, resulting in the PDF of deflection. A sample of the maximum deflection FEM results is shown in Figure. 6. It is noted that the maximum deflection occurs nearly at the middle of the UBM wall. Maximum deflection decreases with thickness as shown in Figure. 7. However, the effect of weighted strength becomes larger as the wall thickness gets smaller. In Figure. 8, maximum deflection increases with pressure. In the mean time, the effect of strength becomes larger as the pressure gets higher.

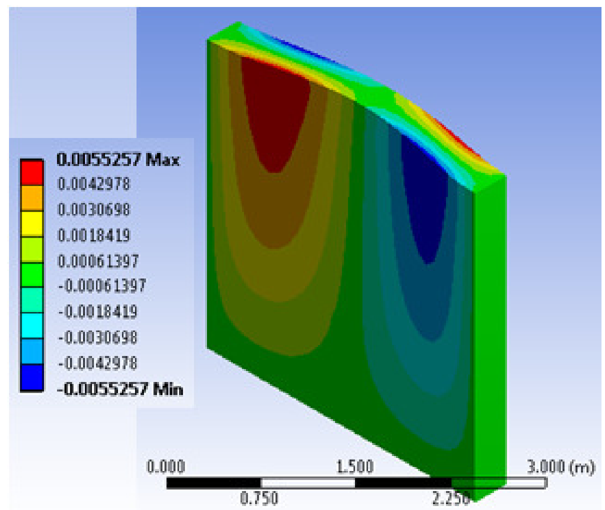

Figure 6. Maximum deflection in FEM

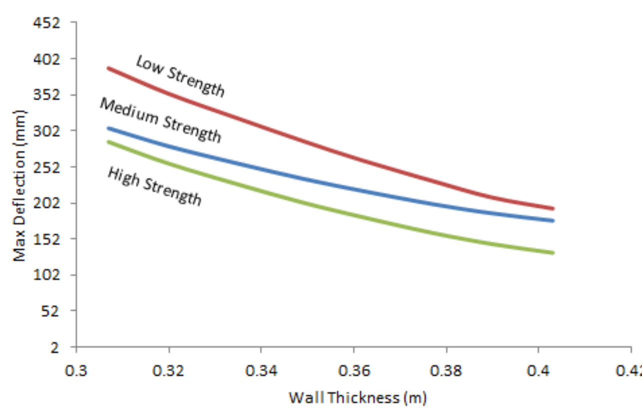

Figure 7. Maximum deflection vs. Thickness (m)

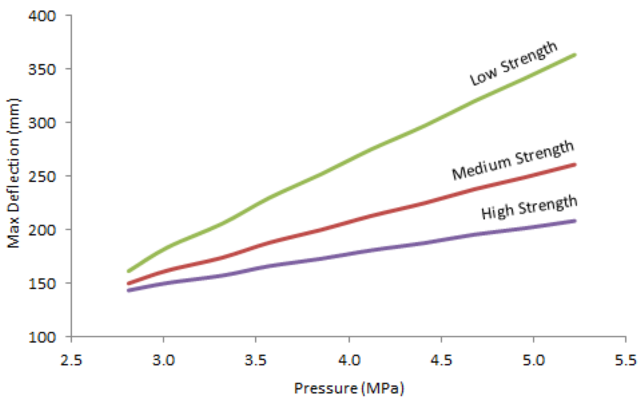

Figure 8. Maximum deflection vs. pressure

5.2. MCS of the Blast Load Model

MCS is conducted using ANSYS/DesignXplorer® in conjunction with explicit FEM using LS-DYNA. Blast loading is calculated based on ConWep [23] by defining the weight of explosive (100 kg) and stand-off distance (20 m) to simulate a surface burst. The FEM input parameters are simulated as shown in Figure. 1. 3 cases are generated from each distribution using LHC sampling technique. Once these cases are fully iterated, 27 responses are obtained based on the generated random input samples. The results are plotted in Figure. 9, where there is a small probability that maximum deflection exceeds wall thickness. Therefore, the occurrence of total wall collapse is a remote possibility [10]. However, a maximum deflection less than wall thickness may cause some degree of damage to the wall. These other –less than total-damage extents include intermediate (non-reusable) and minor (reusable) damages. The probability of any of these scenarios can be directly calculated using the PDF function of maximum deflection (shown in (9)).

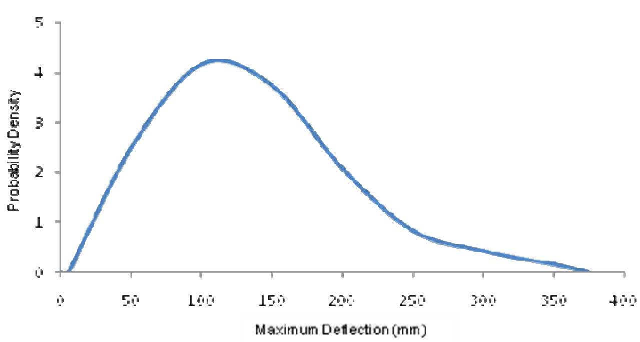

Figure 9. PDF of Maximum deflection

The PDF function of maximum deflection is obtained by performing a regression analysis for the corresponding data. The regression curve of the PDF of maximum deflection (Figure. 9) is represented by the equation:

$$p(x) = -8.07 \times 10^{-8} x^4 + 1.02154 \times 10^{-4} x^3 - 0.0423 x^2 + 5.7984x - 18.2006 \quad (9)$$

By integarating (9), the Cumulative Distribution Function (CDF) of maximum deflection is obtained and plotted in Figure. 10. The CDF is defined as:

$$P(x) = -1.614 \times 10^{-8} x^5 + 2.553 \times 10^{-5} x^4 - 0.0141 x^3 + 2.8992 x^2 - 18.2006 x \quad (10)$$

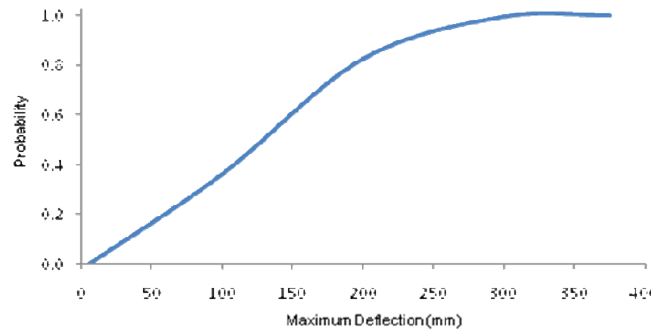

Figure 10. CDF of maximum deflection

Finally, by calculating the reliability at various deflections, the survival function is obtained as shown in Figure. 11. From the figure, it is noted that the reliability stays constant until it drops when the deflection comes closer to the wall thickness. If the definition of reliability is changed to less than total collapse, the shape of the curve will be diffirent. It will actually drop earleir than it does here.

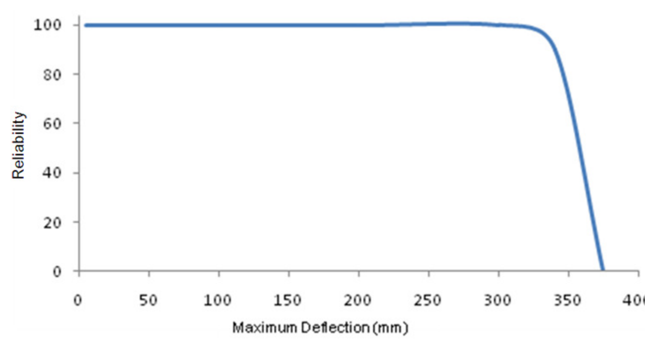

Figure 11. Survival function

VI. CONCLUSIONS

A stochastic simulation is conducted to evaluate the reliability of UBM wall subjected to blast load. The resulting probability of failure indicates that the UBM wall has a high reliability in case of blast loading. The sensitivity analysis indicated that the major factors affecting wall deflection and hence reliability are wall thickness, blast pressure, bond strength, as well as Brick and Mortar strengths. However, weighted averages of these factors are calculated in order to reduce the computational cost. In the mean time, if more computational power is available, it is better to use the full detailed model. Charge weight and stand-off distance can be represented by dynamic pressure. Likewise, the uncertainties of charge weight and stand-off distance can be represented by uncertainties of dynamic pressure affecting the wall. The resulting maximum deflection decreases with thickness. However, the effect of weighted strength becomes larger as the wall thickness gets smaller. In addition, maximum deflection increases with pressure, but the effect of strength becomes larger as the pressure goes higher.

There is a small probability that maximum deflection exceeds wall thickness. Therefore, the occurrence of total wall collapse is a low possibility. However, a maximum deflection less than wall thickness may cause partial damage to the wall. The probability of partial damage scenarios can be directly calculated using the CDF function of maximum deflection. In the survival function, the reliability stays constant until it drops when the deflection comes closer to the wall thickness. If the definition of reliability is changed to partial damage, the curve will drop down earleir, depending on the new value of deflection.

ACKNOWLEDGMENT

The authors would like to thank the Australian Research Council for their support of this research.

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