

# PREDICTION AND CORRELATIONS OF RESIDUAL ENTROPY OF SUPERHEATED VAPOR FOR PURE COMPOUNDS

*Prof. Dr. Mahmoud O. Abdullah<sup>1</sup>, Dr. Sarmad T. Najim<sup>2</sup>, Assist. Lect. Shahad Z. Atta<sup>3</sup>*

Chemical Eng. Department, college of engineering, Nahrain University<sup>1,2,3</sup>

[dr.sarmad\\_alani@yahoo.com](mailto:dr.sarmad_alani@yahoo.com), [shahad\\_zs@yahoo.com](mailto:shahad_zs@yahoo.com)

## Abstract

The calculation of the entropy (S) for gases is essential for design process equipments which involve the calculation of the heat and work requirements for industrial processes. Prediction of accurate values of residual entropy ( $S^R$ ) is necessary step for the calculation of the entropy. In order to obtain accurate values of ( $S^R$ ), attention has been oriented to calculate it using an Equation Of State (EOS). In this paper important equations of state as Lee-Kesler and Virial equation truncated to second and to third terms were used to calculate the residual entropy for pure superheated vapor. In addition to those equations, cubic equations of state represented by Soave-Redlich-Kwong (SRK), and Peng-Robinson equations were also used. These EOS were tested for the available 2791 experimental data points of 20 pure superheated vapor compounds (14 pure nonpolar compounds + 6 pure polar compounds). The Average Absolute Deviation (AAD) for  $S^R$  of 2791 experimental data points of the all 20 pure compounds (nonpolar and polar) when using equations of Lee-Kesler, Peng-Robinson, Virial truncated to second and to third terms, and Soave-Redlich-Kwong were 4.0591, 4.5849, 4.9686, 5.0350, and 4.3084 J/mol.K respectively. It was found from these results that the Lee-Kesler equation was the best (more accurate) one compared with the others, but this equation is sometimes not very preferable, because it needs more time and not easy to apply as cubic equations of state. Noted that SRK equation was the closest one in its accuracy to that of the Lee-Kesler equation in calculating the residual entropy  $S^R$  of superheated vapor, but it was developed primarily for calculating vapor-liquid equilibrium and to overcome this problem, efforts were directed toward possibility of modifying SRK equation to increase its accuracy in predicting the residual entropy as much as possible. The modification was made by redefining the parameter  $\alpha$  in SRK equation to be a function of reduced pressure, acentric factor, and polarity factor for polar compounds in addition to be originally function of reduced temperature and n parameter –which is also function of acentric factor– by using statistical methods. This correlation is as follows:

$$\alpha = [1 + n(\gamma)]^2, \quad \gamma = -0.920338 P_r^{-0.34091} + 0.064049 T_r^4 \omega + 0.370002 \omega - P_r^{0.996932} T_r^{-4} \chi$$

This new modified correlation decreases the deviations in results obtained by using SRK equation in calculating  $S^R$  when comparing with the experimental data. The AAD for 2791 experimental data points of 20 pure compounds is 4.3084 J/mol.K while it becomes 2.4621 J/mol.K after modification. Thus SRK equation after this modification gives more accurate results for residual entropy of superheated vapor of pure 20 compounds than the rest of equations mentioned above.

**Keywords:** entropy, residual entropy, superheated vapor, equation of state, reduced temperature, reduced pressure, acentric factor, and polarity factor,

## 1. Introduction

Thermodynamics has been called by many “the science of energy and entropy”. However, unlike energy, the word entropy is seldom heard in everyday conversation, energy and entropy play important roles in thermal systems engineering [1].

Although there are many ways to introduce the concept of entropy, the simplest is just to deal with its utility; namely: a mathematical tool to describe the direction in which things actually occur and if it occurs spontaneously or not, and it is one of the thermodynamic properties of fluids that are essential for design process equipment that calculates the heat and work requirements of industrial process, and also the analysis of the performance of compressors or expanders requires knowledge of the entropy behavior. Neither energy nor entropy can be measured directly on energy or entropy meter, so values are usually expressed in relation to an

arbitrary reference state by depending on the experimental data of another property that can be measured experimentally as Temperature and Pressure that denoted by **T** and **P** respectively [2, 3].

One of the important ways to obtain the entropy data for pure substances at various states is the experimental data usually available in graphical or tabular forms, but for graphical it is a more complicated method to practical use comparing with the property tables that provide very accurate information about the properties simply, but they are very bulky and vulnerable to typographical errors [2] and many times some interpolations between two pressures or temperatures is needed to obtain the value of a thermodynamic property as entropy at certain point. Also for the phase of superheated vapor it is so difficult reaching the conditions

of high pressures or temperatures for many compounds in laboratory, thus a more practical and desirable approach a relation based on **Equation Of State (EOS)**.

Pure gases are categorized into [4]:

**1. Nonpolar gases** which include:

- a. Simple fluids with spherical molecules  $\omega=0$  as Argon, Krypton,
- b. Quantum gases having  $\omega<0$  as He, H<sub>2</sub>, and
- c. Other nonpolar fluids which are having  $\omega>0$  as Benzene, Propane.

**2. Polar gases** that could be subdivided into:

- a. Non-hydrogen bonding compounds such as Ketones, and Aldehydes, and
- b. Hydrogen bonding compounds (a bond forms between the H atom attached to Oxygen atom in one molecule with the Oxygen atom of another molecule) such as Alcohols, and Water.

In addition to acentricity, the polar compounds are characterized by the presence of dipole moment arises from positive and negative charges that are present in the molecule.

There is no precise recommended method for calculating entropy or residual entropy for superheated vapor. This work involves studying the deviation in calculated entropy values from its actual values (obtained by available experimental data for different compounds: polar and nonpolar gases) then stating which method is more suitable than the others.

## 2. EOS (Models)

The most convenient method of representing the properties or the behavior of a substance, is by a mathematical expression; that is, an equation which represent the (P-V-T) behavior of a fluid. A general form of such an expression known as EOS is:

$$f(P, V, T) = 0 \quad (1)$$

### 2.1 Soave-Redlich-Kwong (SRK) Equation

Soave [5, 6] in 1972 introduced a modification on the Redlich-Kwong (RK) equation of state ; this modification has been successful in extending the applicability of RK equation to be applied with high accuracy for wide range of non-polar and slightly polar components. The temperature dependent term  $a/T^{0.5}$  of

the RK equation was altered to include both the temperature and the acentric factor by Soave; the SRK equation is [7]:

$$P = \frac{RT}{V-b} - \frac{a\alpha}{V(V+b)} \quad (2)$$

Where the factor  $\alpha$  is an empirical function determined from the vapor-pressure data of pure hydrocarbons.

The SRK equation for residual entropy is:

$$\frac{S^R}{R} = \ln[Z-B] - \frac{BD}{a\alpha A} \ln\left(1 + \frac{B}{Z}\right) \quad (3)$$

The cubic form in terms of compressibility factor is:

$$Z^3 - (1-B)Z^2 + (A - B^2 - B)Z - AB = 0 \quad (4)$$

Where

$$A = 0.42747\alpha \frac{P_r}{T_r^2} \quad B = 0.08664 \frac{P_r}{T_r} \quad (5)$$

$$a = 0.42747 \frac{R^2 T_c^2}{P_c} \quad b = 0.08664 \frac{RT_c}{P_c} \quad (6)$$

$$\alpha = \left[1 + n(1 - T_r^{0.5})\right]^2 \quad (7a)$$

$$n = 0.48508 + 1.55171\omega - 0.1561\omega^2 \quad (7b)$$

$$\text{and} \quad D = na \sqrt{\alpha T_r} \quad (8)$$

### 2.2 Peng-Robinson (PR) Equation

The equation of Peng and Robinson in 1976 [8] is structurally rather similar to the SRK and, like the SRK, requires only the critical constants and the acentric factor for its application for a pure fluid. This equation of state was developed primarily for vapor liquid equilibrium predictions. Peng-Robinson modified the standard form as follows [7]:

$$P = \frac{RT}{V-b} - \frac{a\alpha}{V(V+b)+b(V-b)} \quad (9)$$

The SRK equation for residual entropy is:

$$\frac{S^R}{R} = \ln[Z-B] - \frac{BD}{2.828 a\alpha A} \ln\left(\frac{Z+2.414B}{Z-0.414B}\right) \quad (10)$$

The cubic form in terms of compressibility factor is:

$$Z^3 - (1-B)Z^2 + (A - 3B^2 - 2B)Z - (AB - B^2 - B^3) = 0 \quad (11)$$

$$\text{Where} \quad A = 0.45724\alpha \frac{P_r}{T_r^2} \quad B = 0.07780 \frac{P_r}{T_r} \quad (12)$$

$$a = 0.45724 \frac{R^2 T_c^2}{P_c} \quad b = 0.07780 \frac{RT_c}{P_c} \quad (13)$$

$$\alpha = \left[1 + n(1 - T_r^{0.5})\right]^2 \quad (14)$$

$$n = 0.37464 + 1.5422\omega - 0.26992\omega^2 \quad (14)$$

$$\text{also, } D = na\sqrt{\alpha T_r} \quad (15)$$

### 2.3 Lee-Kesler Equation

Lee and Kesler in (1975) [9] developed an analytical correlation, based on Pitzer's three-parameter corresponding states principle [10] to provide increased accuracy and covering the whole range of  $T_r$  and  $Pr$  of practical interest in hydrocarbon processing. It is to be noted that the original correlations by Pitzer et al., were limited to reduced temperatures above 0.8. Pitzer et al. correlations for the compressibility factor of a fluid whose acentric factor is  $\omega$  is given by the following equation:

$$Z = Z^{(0)} + \omega Z^{(1)} \quad (16)$$

Where  $Z^{(0)}$  is the compressibility factor of a simple fluid and  $Z^{(1)}$  corrects  $Z^{(0)}$  for the effects of nonspherical intermolecular forces (primarily dispersion and overlap).  $Z^{(0)}$  and  $Z^{(1)}$  are assumed functions of  $T_r$  and  $Pr$ . However, Lee and Kesler found that the compressibility factor of any fluid is a function of the compressibility of a simple fluid ( $Z^{(0)}$ ), the compressibility of a reference fluid ( $Z^{(r)}$ ), and the acentric factor, where  $Z^{(0)}$  and  $Z^{(r)}$  are functions of  $T_r$  and  $Pr$  and the correlation of Lee and Kesler takes the form:

$$Z = Z^{(0)} + \frac{\omega}{\omega_r} (Z^{(r)} - Z^{(0)}) \quad (17)$$

Where  $\omega_r = 0.3978$  and it is the acentric factor for reference fluid, and the correction term  $Z^{(1)}$  in eq. (16) is obviously equivalent to  $(Z^{(r)} - Z^{(0)}) / \omega_r$  this expression is convenient since both  $Z^{(r)}$  and  $Z^{(0)}$  are given by the same equation with, however, different constants. Lee and Kesler chose n-octane as the heavy reference fluid since it is the heaviest hydrocarbon for which there are accurate (P-V-T) and enthalpy data over a wide range of conditions [9, 11].

The function for both the simple fluid  $Z^{(0)}$  and the reference fluid  $Z^{(r)}$  are derived through a combination of experimental data and a reduced form of the modified Benedict-Webb-Rubin as cited in [9, and 11 ref.] equation of state with a different set of constants that are scheduled in table 1.

$$Z = \left( \frac{P_r V_r}{T_r} \right) = 1 + \frac{B}{V_r} + \frac{C}{V_r^2} + \frac{D}{V_r^5} + \frac{c_4}{T_r^3 V_r^2} \left( \beta + \frac{\gamma}{V_r^2} \right) \exp \left( - \frac{\gamma}{V_r^2} \right) \quad (18)$$

**Table 1** Constants for Calculating equation (18) [9]

constants	Simple Fluid (0)	Reference Fluid (r)
$b_1$		
$b_2$	0.1181193	0.2026579
$b_3$	0.265729	0.331511
$b_4$	0.15479	0.027655
$c_1$	0.030323	0.203488
$c_2$	0.0236744	0.0313385
$c_3$	0.0186984	0.0503618
$c_4$	0.0	0.016901
$d_1 \times 10^4$	0.042724	0.041577
$d_2 \times 10^4$	0.155488	0.48736
$\beta$	0.623689	0.0740336
$\gamma$	0.65392	1.226
	0.060167	0.03754

$$B = b_1 - \frac{b_2}{T_r} - \frac{b_3}{T_r^2} - \frac{b_4}{T_r^3} \quad (19)$$

$$C = c_1 - \frac{c_2}{T_r} + \frac{c_3}{T_r^3} \quad (20)$$

$$D = d_1 + \frac{d_2}{T_r} \quad (21)$$

For calculating  $Z$  for the fluid of interest given at  $T$  and  $P$ , first the appropriate values of  $T_r$  ( $T/T_c$ ) and  $P_r$  ( $P/P_c$ ) are calculating by using critical properties of the fluid. From the simple fluid constants in table 1 and eq. (18) solving for  $V_r$  –which is not the correct reduced volume for the fluid of interest, but rather a pseudo-reduced volume– by the trial and error method when  $V_r$  is defined as  $(P_c V / RT_c)$ , which can be considered the initial guess for the calculation, or from the first equality of eq. (18) the initial guess can be taken as:

$$V_r = Z \frac{T_r}{P_r} \quad (22)$$

the previous equation was depended by Paul and Francis [12] in preparing their computer program for the tables of Lee and Kesler\*. After trial and error calculation the obtained value of  $V_r = V_r^{(0)}$  for simple fluid and when employed in the first equality of eq. (18),  $Z^{(0)}$  is calculated for simple fluid. This process is then repeated using the reference fluid constants with the same  $T_r$  and  $P_r$  values of the fluid of interest to find  $V_r = V_r^{(r)}$  and  $Z^{(r)}$  for the reference fluid. Finally, with  $Z^{(0)}$  from the first calculation

\* Paul and Francis [12] took the initial guess for  $Z$  of eq. (23) equal to 0.2 but in this research which concentrates on the vapor phases especially superheated vapor the initial  $Z$  value taken equal to 1 because for vapor phases at high reduced temperatures and pressures the  $Z$  value often more than unity [13].

and  $Z^{(r)}$  from the second, the compressibility factor  $Z$  for the fluid of interest is determined from eq. (17) [9].

The residual entropy is derived from eq. (18):

$$\frac{S - S^{ig}}{R} + \ln\left(\frac{P}{P^*}\right) = \ln(Z) - \frac{b_1 + \left(\frac{b_3}{T_r}\right) + \left(\frac{2b_4}{T_r^3}\right)}{V_r} - \frac{c_1 - \left(\frac{2c_3}{T_r^3}\right)}{2V_r^2} - \frac{d_1}{5V_r^5} + 2E \quad (23)$$

Where:

$$E = \frac{c_4}{2T_r^3\gamma} \left\{ \beta + 1 - \left( \beta + 1 + \frac{\gamma}{V_r^2} \right) \exp\left( -\frac{\gamma}{V_r^2} \right) \right\} \quad (24)$$

After determining  $V_r^{(0)}$  and  $Z^{(0)}$  for the simple fluid at the  $T_r$  and  $P_r$  appropriate for the fluid of interest, and employing eq. (23) with the simple fluid constants in table 1,  $(S - S^{ig})/R$  is calculated. This term represents  $[(S - S^{ig})/R]^{(0)}$  in this calculation and  $Z$  in eq. (23) is  $Z^{(0)}$ . Then, when repeating the same calculation, using the same  $T_r$  and  $P_r$  and the values of  $V_r^{(r)}$  and  $Z^{(r)}$  for the reference fluid which also determined previously, but employing the reference fluid constants from table 1. With these, eq. (23) allows the calculation of  $[(S - S^{ig})/R]^{(r)}$ . Now, determining the residual entropy function for the fluid of interest from:

$$\left[ \frac{(S - S^{ig})}{R} + \ln\left(\frac{P}{P^*}\right) \right] = \left[ \frac{(S - S^{ig})}{R} + \ln\left(\frac{P}{P^*}\right) \right]^{(0)} + \left( \frac{\omega}{\omega_r} \right) \left\{ \left[ \frac{(S - S^{ig})}{R} + \ln\left(\frac{P}{P^*}\right) \right]^{(r)} - \left[ \frac{(S - S^{ig})}{R} + \ln\left(\frac{P}{P^*}\right) \right]^{(0)} \right\} \quad (25)$$

## 2.4 Virial Equation

The virial equation of state, also called the virial expansion, is the most interesting and versatile of the equations of state which are used to describe the (P-V-T) properties of a fluid and its importance due to that it has a sound theoretical basis. It is a polynomial series in pressure or in inverse volume whose coefficients are functions only of  $T$  for a pure fluid. Virial coefficients are classified into many truncated forms according to the order of the term series. The consistent forms for the initial terms are:

$$Z = \frac{PV}{RT} = 1 + \frac{B(T)}{V} + \frac{C(T)}{V^2} + \frac{D(T)}{V^3} \dots \quad (26a)$$

$$= 1 + B\rho + C\rho^2 + D\rho^3 \dots \quad (26b)$$

$$= 1 + B'P + C'P^2 + D'P^3 \dots \quad (26c)$$

The coefficient  $B$  or  $B'$  is called the second virial coefficient,  $C$  or  $C'$  is called the third virial coefficient, and so on. In practice, since not all of the coefficients of the virial series are known, and only data of the second virial coefficients are plentiful in the literature, terms above the third virial coefficient are rarely used in chemical thermodynamics and the series is usually limited in practice up to moderate pressures. However, the advantages of the virial equation could be increased if quantitative information were available on the third virial coefficient [3, 14, 15].

### 2.4.1 Relations between the virial coefficients

The virial expansion for  $P$  is:

$$P = \frac{RT}{V} \left( 1 + \frac{B(T)}{V} + \frac{C(T)}{V^2} + \frac{D(T)}{V^3} \dots \right) \quad (27)$$

The coefficients of the expansion in pressure are related to the coefficients of the expansion in density ( $1/V$ ) as follows [16]:

$$B = RTB' \Rightarrow B' = \frac{B}{RT} \quad (28a)$$

$$C = (RT)^2(C' + B'^2) \Rightarrow C' = \frac{C - B^2}{(RT)^2} \quad (28b)$$

The first step of the derivation of these relations is by solving the original virial expansion for  $P$  above then equating the two virial expansions, and substituting this expression for  $P$  into the pressure form-side of resulting equation to obtain:

$$1 + B\frac{1}{V} + C\frac{1}{V^2} + \dots = 1 + B'RT\frac{1}{V} + B'RTB\frac{1}{V^2} + C'(RT)^2\frac{1}{V^2} + \dots \quad (29)$$

Both sides of equation (29) are power series in  $1/V$  (third and higher powers of  $1/V$  have been omitted because the second power is the highest power that are using by the common references). Since the two power series must be equal, the coefficients of each power of  $1/V$  must be the same on both sides this comparison provides the relations between the coefficients [15].

### 2.4.2 Second Virial Coefficient

Correlation of second virial coefficient of both polar and nonpolar systems is presented by [4, 15].

$$Z = 1 + \frac{B}{V} = 1 + B'P = 1 + \frac{BP}{RT} \quad (30)$$

Tsonopoulos correlation for  $B$ :

$$B = \frac{RT_c}{P_c} (B^{(0)} + \omega B^{(1)}) \quad (31)$$

$$B^{(0)} = 0.1445 - \frac{0.33}{T_r} - \frac{0.1385}{T_r^2} - \frac{0.0121}{T_r^3} - \frac{0.000607}{T_r^8} \quad (32)$$

$$B^{(1)} = 0.0637 - \frac{0.331}{T_r^2} - \frac{0.423}{T_r^3} - \frac{0.008}{T_r^8} \quad (33)$$

### 2.4.3 Third Virial Coefficient

At high pressures -above 1500 kPa- equations. (26a, b, and c) may be truncated after three terms [13]:

$$Z = 1 + \frac{B}{V} + \frac{C}{V^2} = 1 + B'P + C'P^2 \quad (34)$$

Orbey-Vera correlation for  $C$ :

$$C = \left( \frac{RT_c}{P_c} \right)^2 (C^{(0)} + \omega C^{(1)}) \quad (35)$$

$$C^{(0)} = 0.01407 + \frac{0.02432}{T_r^{2.8}} - \frac{0.00313}{T_r^{10.5}} \quad (36)$$

$$C^{(1)} = -0.02676 + \frac{0.0177}{T_r^{2.8}} + \frac{0.04}{T_r^3} - \frac{0.003}{T_r^6} - \frac{0.00228}{T_r^{10.5}} \quad (37)$$

By using the residual properties, the final expression of the residual entropy after derivation can be expressed as [13]:

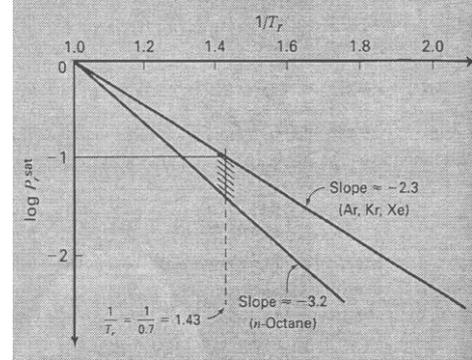
$$\frac{S - S^{ig}}{R} = - \left( \frac{dB}{dT} \left( \frac{P}{R} \right) - \frac{1}{2} \left[ C - T \frac{dC}{dT} - \left( B^2 - 2BT \frac{dB}{dT} \right) \right] \left( \frac{P}{RT} \right)^2 + \dots \right) \quad (38)$$

**The aim of the present work** is to calculate the residual entropy by using (Lee-Kesler, Peng-Robinson, Virial truncated to B or to C terms, and Soave-Redlich-Kwong equations) by determining the deviation from the actual residual entropy using statistical methods to modify the best equation depending on the shape of particle ( $\omega$ ) in addition to the polarity factor ( $\chi$ ) for the polar gases in order to come out with an equation that predicts the residual entropy for different types of superheated vapor of pure gases with high agreement with experimental data.

### 3. Acentric factor

In 1955 Pitzer [10] observed that the reduced vapor pressures of molecules with acentric force fields are

lower than that of simple fluids; and the difference is greater for molecules of greater acentricity. Pitzer noted that all vapor pressure data for the simple fluids (Ar, Kr, and Xe) lie on the same line when plotted as  $\log_{10} P_r^{\text{sat}}$  vs.  $1/T_r$  and that the line passes through  $\log_{10} P_r^{\text{sat}} = -1.0$  at  $T_r = 0.7$ . This is illustrated in Fig. 1.



**Figure 1** Approximate temperature dependence of the reduced vapor pressure [15].

Data for other fluids define other lines whose location can be fixed in relation to the line of simple fluids. Thereupon, Pitzer defined the acentric factor  $\omega$  (a third parameter) of a substance by [15]:

$$\omega = -1 - \log_{10} (P_r^s)_{T_r=0.7} \quad (39)$$

Therefore  $\omega$  can be determined for any fluid from  $T_c$ ,  $P_c$ , and a single vapor-pressure measurement was made at  $T_r = 0.7$  which is near the normal boiling point of most substances, so the importance of choosing  $T_r = 0.7$  that was adopted by Pitzer not only provides numerical simplicity ( $\log_{10} P_r^{\text{sat}} = -1$  for simple fluids) but also beneficial because vapor pressure data are most commonly available at pressure near atmospheric [15].

### 4. Polarity Factor of Halm and Stiel

Because the vapor pressure formed the basis for the definition of the acentric factor, this property was chosen as the starting point for the extension of the normal fluid approach to polar fluids to obtaining polarity factor  $\chi$  which is an empirical parameter for polar substances similar to the acentric factor for normal fluids. The factor  $\chi$  is defined to be zero for normal fluids and can be expressed for polar fluids as follows:

$$\chi = \log P_r^s \Big|_{T_r=0.6} + 1.552 + 1.7\omega \quad (40)$$

Values of  $\chi$  can be obtained from the literatures for some polar compounds [17, 18].

### 5 Calculation of Entropy for Superheated Region

The calculation of entropy for superheated region needing four steps in a calculational path leading from an initial to a final state of a system as obtained in equation below:

$$\Delta S = \Delta S_v + \int_{T_1}^{T_2} C_p^{ig} \frac{dT}{T} - R \ln \frac{P_2}{P_1} + S_2^R - S_1^R \quad (41)$$

Thus, in Fig. 2. the actual path from state 1 to state 2 – the dashed line – is replaced by a four-step calculational path, these steps visualize as the sum of four changes represented by the sequence of isothermal and isobaric steps:

$$S_2 - S_1 = (S_2 - S_2^{ig}) + (S_2^{ig} - S_1^{ig}) + (S_1^{ig} - S_1^v) + (S_1^v - S_1) \quad (42)$$

- **Step 1 → 1<sup>v</sup>:** The transformation of saturated liquid at  $(T_1, P_1)$  to saturated vapor at  $T_1$  and  $P_1$ :

$$S_1^v - S_1 = \Delta S_v$$

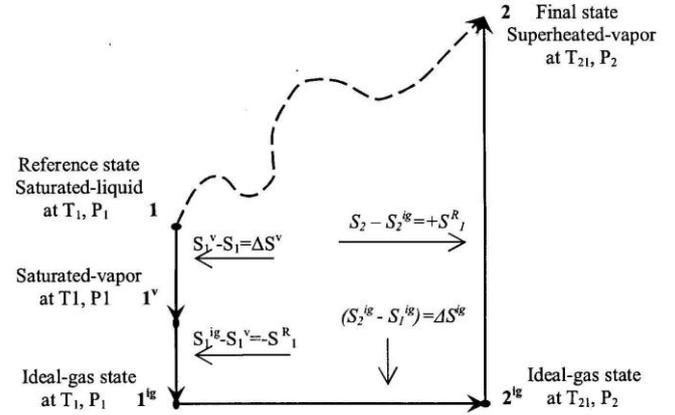
Saturated entropy of vapor can be calculated by converting saturated liquid at reference  $T$  and  $P$  to saturated actual gas at the same  $T$  and  $P$  by using the entropy of vaporization at the normal boiling point  $\Delta S^v$  after scaling it with the reference temperature by Watson relation [15]

$$\frac{\Delta H_{v1}}{\Delta H_{v2}} = \left( \frac{1 - T_{r1}}{1 - T_{r2}} \right)^{0.38} \quad (43)$$

In ref. [13] some of the better estimation methods were tested for several different compounds as hydrocarbons, alcohols, rare gases, oxides and other polar compounds and obtaining useful comparison between these methods (Giacalone, Riedel, Chen, and Vetere) and experimental values of  $\Delta H_v$ . This comparison shows that the average absolute percentage error of Giacalone, Riedel, Chen, and Vetere methods are 2.8, 1.8, 1.7, and 1.6 respectively. Therefore the present investigation employs the more accurate one Vetere method eq. (44) to calculate  $\Delta H^v$  at the normal boiling point and scaled it with eq. (43)

$$\Delta H_{vb} = RT_c T_{br} \frac{0.4343 \ln P_c - 0.69431 + 0.89584 T_{br}}{0.37691 - 0.37306 T_{br} + 0.15075 P_c^{-1} T_{br}^{-2}} \dots \dots \dots (44)$$

to obtain  $\Delta H^v$  at the reference temperature, then calculating  $\Delta S^v$  by dividing  $\Delta H^v$  by the reference temperature.



**Figure 2** Calculational path for entropy change.

- **Step 1<sup>v</sup> → 1<sup>ig</sup>:** A hypothetical process that transforms a real gas into an ideal gas at  $T_1$  and  $P_1$  by using suitable residual entropy of an equation of state.

$$S_{1^{ig}} - S_{1^v} = -S_1^R$$

- **Step 1<sup>ig</sup> → 2<sup>ig</sup>:** Changes in the ideal-gas state from  $(T_1, P_1)$  to  $(T_2, P_2)$ . For this process:

$$\Delta S^{ig} = S_2^{ig} - S_1^{ig} = \int_{T_1}^{T_2} C_p^{ig} \frac{dT}{T} - R \ln \frac{P_2}{P_1}$$

In the present work  $C_p^{ig}$  is calculated by:

$$\frac{C_p^{ig}}{R} = a_0 + a_1 T + a_2 T^2 + a_3 T^3 + a_4 T^4 \quad (45)$$

This polynomial besides it provides simplicity in use, it also covers good range of temperatures. The data related to eq. (45) for some pure compounds are listed in (Appendix A- section c) of ref. [17].

- **Step 2<sup>ig</sup> → 2:** Another hypothetical process that transforms the ideal gas back into a real gas at  $T_2$  and  $P_2$ :

$$S_2 - S_2^{ig} = S_2^R$$

Therefore, Equation (42) is results of the totality of the entropy changes for the above four steps.

### 6. Selecting the Optimum EOS for the Present Work [19]

The conflict between accuracy and simplicity is a big dilemma in the development of an equation of state. Despite the wide use of high-speed computers, the

simplicity is still highly desired for easy and unequivocal applications of the equation to complex problems. So, for most calculations, the empirical approach is often better than the more complex theoretical approach in view of accuracy as well as minimum data requirements. Often, virial equation – which truncates to second or to third term – can only be considered useful from the first form and employed in the present work. In addition to the Soave-Redlich-Kwong equation (SRK) and Peng-Robinson equation (PR) as the models of cubic equations of state from the second form which have valuable applications in most common use today, Lee and Kesler equation represents –although it is including some complexity– the easiest example of the third form that was also used in the present work.

The **A**verage **A**bsolute **D**eviation for residual entropy AAD in (J/mol.K), which is defined as follows:

$$AAD = \frac{\sum |(S_{\text{experimental}}^R - S_{\text{calculated}}^R)|}{n} \quad (46)$$

AAD is considered as a factor for comparison between the different methods that were used for calculating actual residual entropy of superheated vapor for different compounds. It is calculated in (J/mol.K) because when it is calculated in (J/g. K) was being very decimal and the imparity of deviations of using the equations being not perceptible. Also, the dimensionless deviation can be obtained by dividing AAD by R (gas constant) which denoted by AAD/R.

In addition, the **A**verage **A**bsolute **P**ercentage **D**eviation for entropy AAD% is defined as follows:

$$AAD\% = \frac{\sum |(S_{\text{experimental}} - S_{\text{calculated}})/S_{\text{experimental}}| \times 100\%}{n} \quad (47)$$

which is considered as a factor for comparison between the different methods that were associated in determining the actual entropy of superheated vapor for different compounds.

## 7. Results and Discussion [19]

### 7.1 Application of the EOS for Compounds

#### 7.1.1 Classification of the Application of EOS into Regions

Five different equations of state were applied for calculating residual entropy ( $S^R$ ) in comparison for all experimental data of pure compounds that supported the

present investigation as expressed earlier. To be insight of the precision of these equations with the range of  $T_r$  and  $P_r$ , summary of results classified into three regions is presented as follows:

#### Region 1)

$T_r < 1$ , and  $P_r < 1$ : including 14 compounds involving 859 experimental data points (9 nonpolar compounds with 431 experimental data points and 5 polar compounds with 428 experimental data points) from the 2791 experimental data points of the all 20 compounds.

#### Region 2)

$T_r > 1$ , and  $P_r < 1$ : including the 20 compounds involving 1501 experimental data points (14 nonpolar compounds with 921 exp. data points and 6 polar compounds with 580 exp. data points) from the all 2791 exp. data points of the all 20 compounds.

#### Region 3)

$T_r > 1$ , and  $P_r > 1$ : including 13 compounds involving 431 experimental data points (10 nonpolar compounds with 308 experimental data points and 3 polar compounds with 123 experimental data points) from the all 2791 experimental data points of the all 20 compounds. Although this region represents the supercritical region, but the knowledge of the accuracy of employing these equations in comparison with the accuracy of the modified equation in the present work for this region is advantageous for the some available experimental data.

**Table 2** The existence of the compounds in 3regions.

Non polar compounds		regions	Non polar compounds		regions
1	Argon	1, 2, 3	12	n-Hexane	1, 2, 3
2	Methane	2, 3	13	n-Heptane	1, 2, 3
3	Oxygen	2, 3	14	n-Octane	1, 2, 3
4	Nitrogen	2, 3			
5	Ethane	2, 3	Polar compounds		regions
6	Cyclopropane	1, 2	1	Refrigerant 12	1,2
7	Propane	1, 2, 3	2	Isopentane	2, 3
8	Acetylene	1, 2	3	Ammonia	1, 2
9	Neopentane	1, 2	4	Refrigerant 152a	1, 2
10	Benzene	1, 2	5	Refrigerant 134a	1, 2, 3
11	Carbon dioxide	2, 3	6	Water	1, 2, 3

#### 7.1.2 Total Region

This region represents all three regions and consists of 20 compounds involving 2791 experimental data points

(14 nonpolar compounds with 1660 experimental data points and 6 polar compounds with 1131 experimental data points).

## 7.2 Modification of EOS [19]

### 7.2.1 Selecting the Optimum EOS for the Modification

Although Lee-Kesler equation proved to be better than Soave-Redlich-Kwong, Peng-Robinson, and Virial equations for the prediction of residual entropy of superheated vapor for the most of compounds that used in this investigation, but it is more useful if the errors can be reduced to value less than those obtained with Lee-Kesler equation.

The modification of any equation is usually done for the equation that proved to be the most accurate. The more accurate one is Lee-Kesler equation, but it is very difficult to modify it, so Soave-Redlich-Kwong equation which is the nearest one in accuracy to the Lee-Kesler equation was selected for modification.

### 7.2.2 Modification of Soave-Redlich-Kwong Equation [19]

Soave-Redlich-Kwong equation was derived mainly to calculate vapor-liquid equilibria, so all attention was concentrated on that purpose in its derivation, and therefore, there is still room for improving it for superheated vapor.

The modification would be based on modifying  $\alpha$  parameter of Soave equation which is function of reduced temperature and also on acentric factor which is included in parameter  $n$ :

$$\alpha = \left[ 1 + n \left( 1 - T_r^{0.5} \right) \right]^2 \quad \text{Soave eq. parameter} \quad (7a)$$

Figures 2 to 5 show clearly that the values of pressures or reduced pressures influence on the value of  $\alpha$  although the temperature is constant. Thus  $\alpha$  in Soave equation can be considered a function of temperature, pressure and acentric factor and its equation would be written as [19]:

$$\alpha = \left[ 1 + n(\gamma) \right]^2 \quad \text{new form of } \alpha \text{ parameter} \quad (48a)$$

$$\gamma = g_1 Pr^{g_2} + g_3 Tr^4 \omega + g_4 \omega - Pr^{g_5} Tr^{-4} \chi \quad (48b)$$

The coefficients of this equation had been determined by using statistical methods. These coefficients were calculated with the aid of computer program on non-linear

estimation of statistica software that fitting to minimize the error obtained for calculating new  $\alpha$  for two selected compounds.

**Table 3** Coefficients of equation (48b).

coefficient	value	coefficient	value
$g_1$	-0.920338	$g_4$	0.370002
$g_2$	-0.034091	$g_5$	0.9906321
$g_3$	0.064049	----	----

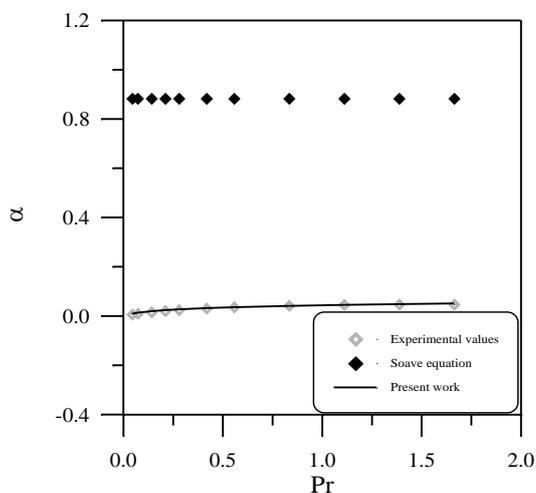
In the present work n-octane and water were used in the fitting. By trying many different equations, it was found that equation (48b) was the optimum equation for predicting  $\gamma$  with lowest error. The coefficients of eq. (48b) are listed in table 3.

The modification was made by comparing the experimental values of residual entropy of superheated vapor with the values calculated by Soave equation which was obtained by using all original parameters of Soave equation except  $\gamma$  parameter which is inserted in the computer program empirically and remain varying until the deviation between the inserted value of  $\gamma$  with the calculated value by Soave equation for each experimental data point- approached to zero (% error of  $(S_{exp}^R - S_{cal}^R) \leq 0.00001$ ).

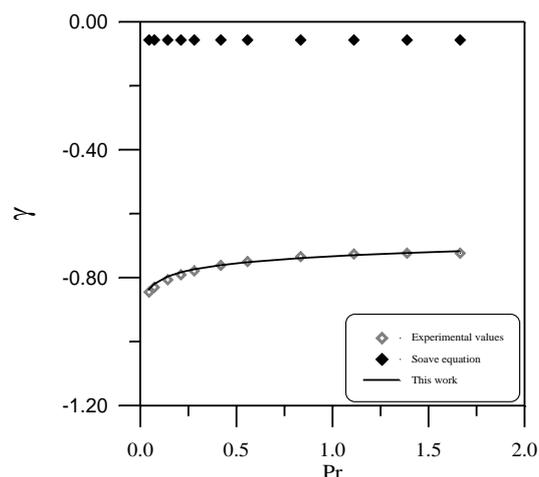
The new equation of  $\alpha$  gives a higher accuracy where the overall **AAD** was 2.4621 J/mol.K for residual entropy and the **(AAD%)** was 1.1083 for entropy, for the all compounds studied in the present work.

The main reason for choosing the mentioned two compounds (n-octane, and water) in the fitting of the experimental data due to their molecular nature that n-octane represents the normal nonpolar gases which was considered having the highest  $\omega > 0$  in comparison with the other used compounds in the present work and thus to be able of controlling the other compounds of less  $\omega$  and has no polarity properties found ( $\chi = 0$ ). On the other hand water represents the polar compounds which have in addition to  $\omega > 0$  the polarity properties  $\chi > 0$  and also it has the highest  $\chi$  among others. The only way for obtaining the experimental values of the parameter  $\gamma$  is by empirically trial and error method which needs great time to obtain any value. Therefore the modification was limited to only these mentioned two types of gases which

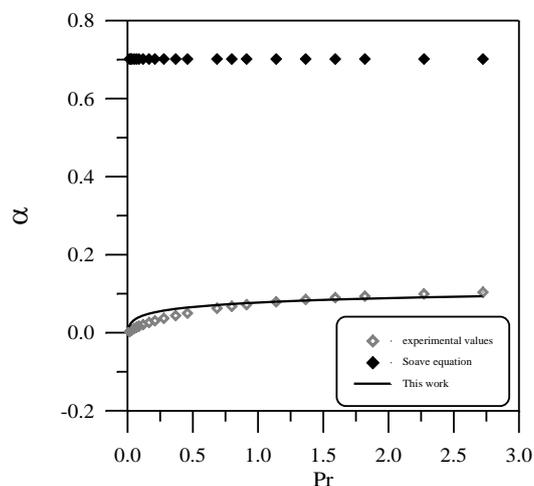
were used to predict the new  $\alpha$  parameter equation that gives more accuracy in calculating  $S^R$  and  $S$  for all data points (2791) experimental data points for the all 20 nonpolar and polar gases employed in the present work.



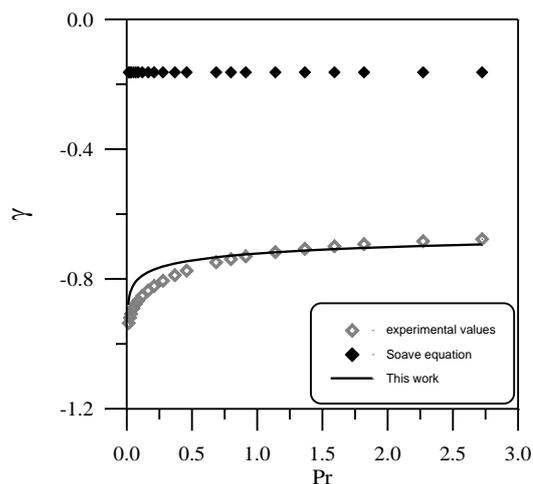
**Figure 2** The relation between the values of parameter  $\alpha$  and  $P_r$  for n-octane at  $T_r = 1.3$



**Figure 3** The relation between the values of parameter  $\gamma$  and  $P_r$  for n-octane at  $T_r = 1.3$



**Figure 4** The relation between the values of parameter  $\alpha$  and  $P_r$  for water at  $T_r = 1.349$



**Figure 5** The relation between the values of parameter  $\gamma$  and  $P_r$  for water at  $T_r = 1.349$

**Table 4a** Summary of application of EOS through 3 regions individually.

Region 1 at ( $T_r < 1$ and $P_r < 1$ ) No. of points in region 1 is (859)			
Equations used	AAD for $S^R$ (J/mol.K)	AAD/R for $S^R$	AAD% for S
L-K	4.8377	0.5819	1.7033
P-R	5.2743	0.6344	1.9060
Virial truncated to B)	5.2816	0.6353	1.9657
Virial truncated to C)	5.2197	0.6278	1.9547
S-R-K	5.1998	0.6254	1.8723
This work	2.9180	0.3510	1.0974

Region 2 at ( $T_r > 1$ and $P_r < 1$ ) No. of points in region 2 is (1501)			
Equations used	AAD for $S^R$ (J/mol.K)	AAD/R for $S^R$	AAD% for S
L-K	3.9300	0.4727	1.7892
P-R	4.1422	0.4982	1.8790
Virial truncated to B)	4.3718	0.5258	2.0988
Virial truncated to C)	4.3709	0.5257	2.1102
S-R-K	4.0144	0.4828	1.8171
This work	2.4077	0.2896	1.1475

Region 3 at ( $T_r > 1$ and $P_r > 1$ ) No. of points in region 3 is (431)			
Equations used	AAD for $S^R$ (J/mol.K)	AAD/R for $S^R$	AAD% for S
L-K	2.9571	0.3557	1.5221
P-R	4.7524	0.5716	2.7529
Virial truncated to B)	6.4233	0.7726	4.6412
Virial truncated to C)	6.9797	0.8395	5.0765
S-R-K	3.5325	0.4249	2.1336
This work	1.7430	0.2096	0.9935

**Table 4b** Summary of application of EOS through whole regions. (from  $T_r < 1$  &  $P_r < 1$  to  $T_r > 1$  &  $P_r > 1$ )

All regions (1, 2, and 3) No. of points for all regions (2791)			
Equations used	AAD for $S^R$ (J/mol.K)	AAD/R for $S^R$	AAD% for S
L-K	4.0591	0.4882	1.7215
P-R	4.5849	0.5515	2.0223
Virial truncated to B)	4.9686	0.5976	2.4505
Virial truncated to C)	5.0350	0.6056	2.5204
S-R-K	4.3084	0.5182	1.8850
This work	2.4621	0.2961	1.1083

## 8. Discussion

The entropy of a pure compound cannot be directly measured but is calculated from other properties. It is a function of both temperature and pressure; in general, it increases with the increase in temperature and decreases with the increase in pressure at constant temperature. At zero pressure, all gases behave ideally, and the ideal gas state entropy is remaining dependent on pressure.

The residual entropy term  $S-S^{ig}$ , is the difference between the entropy of a compound at certain P and T and that of an ideal gas state at the same conditions. In the absence of P-V-T data for the compounds of interest, or if the data do not cover the conditions under which engineering calculations are to be made, generalized correlations which express Z as a function of  $T_r$  and  $P_r$  have been found to be of great value in estimating residual properties as residual entropy which are based on a modified theory of corresponding states.

The usual method available for predicting the residual entropy of superheated vapor for pure compounds is by employing the equation of state. In the present work the equations of state employed were: L-K, P-R, S-R-K, and Virial truncated to second and to third terms equations. It is well known that the evaluation of any correlation or prediction method is done by comparison of the calculated values for any equation used with those of the experimental data which normally limited and mostly not covering wide range of temperatures and pressures for any certain compound. The deviation between the experimental data and results of prediction or correlation method determines the accuracy of the method and this accuracy in the present work was represented as

mentioned in the previous section by AAD% for entropy and AAD J/mol.K for residual entropy. Tables 4a and 4b show the AAD% and the AAD in J/mol.K for the calculated entropy and calculated residual entropy respectively of the superheated vapor for nonpolar and polar compounds as compared with experimental values.

### 8.1 Comparison of the Results with the Experimental Data [19]

Comparing the results that are shown in Tables 4a and 4b, indicate that the L-K equation gives higher accuracy for predicting  $S^R$  as compared with the P-R, Virial truncated to second or third terms, and S-R-K equations using the experimental data points of the present study.

The AAD% for calculating the entropy of superheated vapor for 14 nonpolar compounds of 1660 data points are 1.389%, 1.397%, 1.4918%, 1.5237%, and 1.3799% when using L-K, P-R, Virial truncated to second or third terms, S-R-K respectively, while the AAD for calculating the residual entropy of superheated vapor of these compounds by using these equations are 4.6277, 4.9243, 4.9782, 4.9501, and 4.7665 J/mol.K respectively. Further more the AAD% for calculating the entropy of superheated vapor for 6 polar compounds of 1131 data points are 2.2096%, 2.9399%, 3.8575%, 3.9833, and 2.6263% when using L-K, P-R, Virial truncated to second or third terms, S-R-K respectively, while the AAD for calculating the residual entropy of superheated vapor of these compounds by using these equations are 3.2247, 4.0867, 4.9546, 5.1597, and 3.6359 J/mol.K respectively.

The AAD% for calculating the entropy of superheated vapor for the all compounds of 2791 data points are 1.7215%, 2.0223%, 2.4505%, 2.5204%, and 1.8850% when using L-K, P-R, Virial truncated to second or third terms, S-R-K respectively, while the AAD for calculating the residual entropy of superheated vapor of these compounds by using these equations are 4.0591, 4.5849, 4.9686, 5.0350, and 4.3084 J/mol.K respectively.

### 8.2 The Modified Equation [19]

Although of the better results were obtained by L-K equation, but this equation is sometimes not very preferable, because it needs more time and not easy to apply as cubic equations of state. S-R-K equation showed

also a very good accuracy closest to that of the L-K equation in calculating the residual entropy  $S^R$  of superheated vapor. Thus efforts were directed to modify S-R-K equation to increase its accuracy as much as possible and to be more accurate than the original Soave-Redlich-Kwong and even the L-K equation. This may be done by using a statistical program and statistical methods that give the suitable form of correlation.

Many attempts were done to develop S-R-K equation to this purpose, and the best correlation of this modification was obtained in section 3.6 which proved a very good accuracy for most compounds under study.

The modification was applied for 20 pure compounds of 2791 experimental data points (nonpolar and polar compounds). It reduced the AAD% and AAD of Soave-Redlich-Kwong equation for 1660 experimental data points of 14 nonpolar compounds from 1.3799% to 0.9592% and from 4.7665 to 2.8247 J/mol.K respectively. While it reduced the AAD% and AAD of Soave-Redlich-Kwong equation for 1131 experimental data points of 6 polar compounds from 2.6263% to 1.3270% and from 3.6359 to 1.9299 J/mol.K respectively. On the other hand it reduced the AAD% and AAD of Soave-Redlich-Kwong equation for all the 20 pure compounds under study from 1.8850% to 1.1083% and from 4.3084 to 2.4621 J/mol.K respectively as shown in tables 4-1 to 4-3.

Examples of nonpolar compounds: for octane the AAD% and AAD by using SRK equation were 2.0085% and 12.0160 J/mol.K respectively, while they were 0.3711% and 2.1418 J/mol.K respectively when using the modified SRK equation. For nitrogen the AAD% and AAD by using SRK equation were 0.4308% and 0.7276 J/mol.K respectively, while they were 0.2925% and 0.4956 J/mol.K respectively when using the modified SRK equation.

Examples of polar compounds: for water the AAD% and AAD by using SRK equation were 5.1695% and 6.2356 J/mol.K respectively, while they were 2.4539% and 3.2028 J/mol.K respectively when using the modified SRK equation. For refrigerant (134a) the AAD% and AAD by using SRK equation were 3.0307% and 5.8645 J/mol.K respectively, while they were 1.2397 % and 2.3752 J/mol.K respectively when using the modified SRK equation.

For polar compounds when applying the modified SRK equation without considering polarity effect term the deviation from the experimental data was more than that when considering polarity effect term as shown in Table 3-5. Although this increase was not very significant, but in practical use more accuracy is desirable.

The interesting features of the developed equation in the present work for calculating the residual entropy are:

1. It is a rather simple equation that achieved good results for both nonpolar and polar compounds.
2. It needs only well-known properties of pure compounds ( $T_c$ ,  $P_c$ ,  $\omega$ , and for polar compounds  $\chi$ ) for each compound.

The new correlation gives very good accuracy for calculating  $S^R$  of the compounds shown in Table 4-3 by comparison with the experimental data approximately over the whole temperature and pressure ranges tested. Where the conditions tested for temperature up to  $T_r > 2$  and for pressure raised for some compounds up to  $P_r > 2$ . Figures 2 to 5 show the relation either between residual entropy with pressure at constant temperature or between residual entropy with temperature at constant pressure for the results obtained using this new method of correlation and other equations used comparing with the experimental data.

Tables 4a and 4b show the comparison of results of deviations from the experimental data of n-octane and water when using Lee-Kesler, Peng-Robinson, Soave-Redlich-Kwong equations of state, and the new correlation of this work. Although virial equation was employed in this study but its results showed a high deviations from the experimental values obtained comparing with deviations of other equations that employed.

## 9. Conclusions

1. Different equations of state were used to predict the residual entropy of superheated vapor for pure compounds. They are Lee-Kesler, Peng-Robinson, Soave-Redlich-Kwong and Virial truncated to second and to third terms equations of state. The results indicate that Lee-Kesler equation is the most accurate equation among these five equations, SRK equation is the closest one in

its accuracy to the Lee-Kesler, and the virial equation (truncated to second or to third terms) gave highest deviations from the experimental values which proscribed the need to listing its results in tables.

2. New modification was made by redefining the parameter  $\alpha$  in Soave equation to be a function of reduced pressure, acentric factor, and polarity factor for polar compounds in addition to be originally function of reduced temperature and n parameter –which is also function of acentric factor– by using statistical methods. This correlation is as follows:

$$\alpha = [1 + n(\gamma)]^2$$

$$\gamma = -0.920338 P_r^{-0.34091} + 0.064049 T_r^4 \omega - 0.370002 \omega - P_r^{0.996932} T_r^{-4} \chi$$

The results of this correlation were compared with the results obtained from Lee-Kesler, Peng-Robinson, Virial truncated to second and to third terms, and Soave-Redlich-Kwong methods.

**a.** The AAD of 1660 experimental data points of 14 pure nonpolar compounds obtained from this correlation was 2.8247 J/mol.K in comparison with those obtained from Lee-Kesler, Peng-Robinson, Virial truncated to second and to third terms, and Soave-Redlich-Kwong methods were 4.6277, 4.9243, 4.9782, 4.9501, and 4.7665 J/mol.K respectively.

**b.** The AAD of 1131 experimental data points of 6 pure polar compounds obtained from this correlation was 1.9299 J/mol.K in comparison with those obtained when using the same equations above were 3.2247, 4.0867, 4.9546, 5.1597, and 3.6359 J/mol.K respectively.

**c.** The AAD of 2791 experimental data points of all the 20 pure compounds obtained from this correlation was 2.4621 J/mol.K in comparison with those obtained from Lee-Kesler, Peng-Robinson, Virial truncated to second and to third terms, and Soave-Redlich-Kwong methods were 4.0591, 4.5849, 4.9686, 5.0350, and 4.3084 J/mol.K respectively.

### **Acknowledgement**

*To all who try to understand the philosophy of entropy and interpret why it intrudes in every thing of our lifes.*

## **Nomenclature**

### **Variable Notations**

A, B, Constants used in the cubic EOS, eq.(4) and eq. (11), B, B', Second Virial Coefficient, cm<sup>3</sup>/mol<sup>2</sup>, b, c, d, Coefficients of eq.(23), C, C', Third Virial Coefficient, Cm<sup>6</sup>/mol<sup>3</sup>, C<sub>p</sub>, Heat capacity at constant pressure, J/mol.K, g<sub>1</sub>,g<sub>2</sub>,..g<sub>5</sub>, Coefficients of eq. (48b), n, Constant used in the cubic EOS, eq.(4) and eq. (11), P, Pressure, kPa, R, Universal gas constant, J/mol.K., S, Entropy, J/mol.K, T, Temperature, K, V, Volume,m<sup>3</sup>, Z, Compressibility factor.

### **Abbreviations**

AD, absolute deviations for S<sup>R</sup>, J/(mol.K), AD%,absolute percentage deviation for S, AD/R, absolute deviation for S<sup>R</sup> divided by R, AAD, Average Absolute Deviation for S<sup>R</sup>,J/(mol.K) AAD%, Average Absolute Percentage Deviation for S, AAD/R, Average Absolute Deviation per R for S<sup>R</sup>, EOS, Equation Of State, LK, Lee-Kesler equation, M.wt., Molecular weight, PR Peng-Robinson equation, RK, Redlich-Kwong equation, Ref., Reference, SRK, Soave-Redlich-Kwong equation.

### **Greek Letters**

$\alpha$ , Constant Used in cubic equations of state,  $\beta$ , Parameter used in equation (2-57).,  $\gamma$ , Constant used in equation (18),  $\gamma$ , Parameter used in equation (48a),  $\omega$ , Acentric factor.,  $\chi$ , Polarity factor of eq. (40).

### **Superscripts**

ig, Ideal gas, R, Residual, sat., Saturated state, (0), Simple Fluid Equation (17), (r), Reference Fluid Equation (17).

### **Subscripts**

b, Boiling point, c, Critical property, cal., Calculated value, exp.,=,Experimental value, g, Gas state, l, Liquid state, r, Reduced property r, Reference fluid, v, vaporization.

## References

1. Moran, M. J., Shapiro, H. N., Munson, B. R., and De Witt, D. P., "Introduction to Thermal Systems Engineering" John Wiley and Sons, Inc., (2003).
2. Cengel, Y. A., and Boles, M. A., "Thermodynamics: An Engineering Approach" 3<sup>rd</sup> Edition, Mc Graw Hill, (1998).
3. Moran, M. J., and Shapiro, H. N., "Fundamentals of Engineering thermodynamics" 4<sup>th</sup> Edition, John Wiley and Sons, Inc., (2000).
4. Tsonopoulos, C., "An Empirical Correlation of the Second Virial Coefficients" AIChE J., Vol. 20, No. 2, (1974): 263-272.
5. Soave, G., "Equilibrium Constants from a Modified Redlich-Kwong Equation of State" Chem. Eng. Sci., Vol. 27, (1972) pp. 1197-1203.
6. Soave, G., "Improvement of the Van Der Waals Equation of State" Chem. Eng. Sci., Vol. 39, No. 2, (1983) pp. 357-369.
7. Assael, M. J., Trusler, J. P. M., and Tsolakis, T. F., "Thermophysical Properties of Fluids" (1996).
8. Peng, D. Y., Robinson, D. B., "A New Two-Constant Equation of State" Ind. Eng. Chem. Fundam., Vol. 15, 59 (1976).
9. Lee, B. I., and Kesler, M. G., "A Generalized Thermodynamic Correlation Based on Three-Parameter Corresponding States" AIChE J., Vol. 21, No. 3, (1975): 510-527.
10. Pitzer, K. S., Lipmann, D. Z., Curl, R. F., JR., Huggins, C. M., and Petersen, D. E., "The Volumetric and Thermodynamic Properties of Fluids - II. Compressibility Factor, Vapor Pressure and Entropy of Vaporization" American Chemical Society, 77, 3433 (1955)
11. Edmister, W. C., and Lee, B. IK., "Applied Hydrocarbon Thermodynamics" Vol. 1, 2<sup>nd</sup> Edition, Gulf Publishing Company (1984).
12. Paul, B. J., and Francis, O., "Computer Aided Chemical Thermodynamic of Gases and Liquids – Models Programs" John Wily and Sons (1985).
13. Poling, B. E., Prausnitz, J. M., and O'Connell, J. P., "The Properties of Gases and Liquids" 5th Edition, McGraw-Hill (2001).
14. Santis, R. D., and Grande, B., "An Equation for Predicting Third Virial Coefficients of Nonpolar Gases" AIChE J., Vol. 25, No. 6, (1979): 931-938.
15. Smith, J. M., Van Ness, H. C., and Abbott, M. M., "Introduction to Chemical Engineering Thermodynamics" 6<sup>th</sup> Edition, McGraw-Hill (2001).
16. Eubank, P. T., and Angus, S., "Truncation of Virial Equations in PVT Data Reduction" Journal of Chemical and Engineering Data, Vol. 18, No. 4, (1973): 428-430.
17. Halm, L. R., and Stiel, L. I., "A Fourth Parameter for the Vapor Pressure and Entropy of Vaporization of Polar Fluids" AIChE J., 13, 351 (1967).
18. Kukoljac, M. D., and Grozdanic, D. K., "New Values of the Polarity Factor" J. Serb. Chem. Soc., Vol. 65, 12, (2000): 899-904.
19. AL-Najjar, Sh. Z., "Prediction And Correlations Of Residual Entropy Of Superheated Vapor For Pure Compounds" M. Sc. Thesis, Nahrain University (2009).

## تنبؤ وعلاقات عامة لقيمة الإنتروبي

### المتبقي للبخار المحمص للمركبات النقية

أ.د. محمود عمر عبد الله<sup>1</sup>، د. سرمد طالب نجم<sup>2</sup>، م.م. شهد زهير عطا<sup>3</sup>

جامعة النهريين، كلية الهندسة، قسم الهندسة الكيماوية<sup>1,2,3</sup>

#### الخلاصة

ان التنبؤ بالقيمة الصحيحة لمقدار الإنتروبي (entropy) أساسي في عمليات تصميم المعدات، حيث أنها تسهم في تحديد متطلبات الحرارة و الشغل للعملية الصناعية وتمثل معرفة قيمة الإنتروبي المتبقي خطوة مهمة جداً في حساب قيمة الإنتروبي للغازات. بما أن القيم المختبرية لدالة العشوائية للأبخرة المحمصية محدودة وللبعض المركبات والغازات الشائعة فقط مع مدى محدود لدرجات الحرارة والضغط، لذلك قد توجه الاهتمام الى حسابها باستخدام معادلات الحالة.

في هذه الدراسة تم استخدام معادلات مهمة لإحساب قيمة الإنتروبي المتبقي للأبخرة المحمصية النقية منها معادلات Lee-Kesler، Virial المقطوعة لحدين مرة ولثلاثة حدود مرة. بالإضافة الى استخدام المعادلات التكعيبية للحالة المتمثلة بمعادلتين: Soave-Redlich-Kwong و Peng-Robinson. جريت هذه المعادلات على 2791 نقطة مختبرية للبخار المحمص لعشرين مركب نقي (14 مركب نقي غير قطبي و 6 مركب نقي قطبي). كان المعدل المطلق للانحراف للعشرين مركب ككل مع 2791 نقطة مختبرية لتلك المعادلات هو، 4.5849 ، 4.0591 ، 4.9686 ، 5.0350 ، و 4.3084 جول/مول. كلّفن لمقدار الإنتروبي المتبقي على التوالي.

قد وجد من تلك النتائج ان معادلة Lee-Kesler هي الأقل انحراف عن النتائج المختبرية عند مقارنتها ببقية المعادلات لكنها غير مفضلة الاستخدام في أحيان كثيرة لحاجتها الى وقت أكثر من غيرها بالإضافة إلى كونها ليست سهلة في التطبيق كما هو الحال في المعادلات التكعيبية للحالة. بعد ملاحظة ان معادلة SRK كانت هي الأقرب في دقتها لمعادلة Lee-Kesler في حساب مقدار الإنتروبي المتبقي للأبخرة المحمصية، لكنها طورت اساساً للحسابات المتعلقة بالتوازن بين حالي البخار والسائل لذلك وللتغلب على هذه المسألة، بذلت المحاولات باتجاه إمكانية تعديلها لزيادة دقتها في حساب مقدار الإنتروبي المتبقي للأبخرة المحمصية قدر الإمكان.

تم تعديل معادلة SRK بإعادة تعريف المتغير  $\alpha$  في المعادلة ليكون دالة للضغط المنقوص وكذلك عامل القطبية للمركبات القطبية بالإضافة لكونه أساساً دالة لدرجة الحرارة المنقوصة وعامل اللامركزية المضمن في المتغير  $n$  لهذه المعادلة باستخدام الطرق الإحصائية كما يلي:

$$\alpha = [1 + n(\gamma)]^2, \quad \gamma = -0.920338 P_r^{-0.34091} + 0.064049 T_r^4 \omega - 0.370002 \omega - P_r^{0.996932} T_r^{-4} \chi$$

هذه العلاقة الجديدة المعدلة قللت الانحرافات في نتائج معادلة Soave عند مقارنتها بالنتائج المختبرية حيث ان المعدل المطلق للانحراف لعشرين مركب نقي ككل مع 2791 نقطة مختبرية قبل التعديل هو 4.3084 جول/مول. كلّفن بينما اصبح 2.4621 جول/مول. كلّفن لقيمة الإنتروبي المتبقي مع التعديل. بذلك تعطي معادلة SRK بعد هذا التعديل نتائج أكثر دقة لحساب قيمة الإنتروبي المتبقي للأبخرة المحمصية للمركبات النقية من باقي المعادلات المذكورة.

