THE DETECTION OF SHIFTS IN AUTOCORRELATED PROCESSES WITH *MR* AND *EWMA* CHARTS

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ABSTRACT

Since the performance of SPC is known to be seriously deteriorated because of autocorralated observations, the detection of an assignable cause is a critical task that most industrial practitioners have to deal with. For this reason, selecting the most appropriate control chart to separate a shift among autocorrelated observations is a serious problem which needs a thoughtful judgement. In this research, two subclasses of ARIMA models, e.g., AR (1) and IMA (1, 1), were deployed to characterize autocorrelated processes which were categorized into two cases, stationary and non-stationary. The simulation was done to assess how each type of control chart responded to a shift in the form of average run length (ARL) while the factorial analysis was conducted to quantify the impacts of critical factors e.g., AR coefficient (ϕ), MA coefficient (θ), types of charts and shift sizes on the ARL. For non-stationary case, when shift sizes were small ($\delta=0.5\sigma_a$), the ARL at $\theta = +1$ was significantly higher than the one at $\theta = -1$. However, when the observations were stationary, the above result was valid only when an MR chart was utilized. Another significant finding is that the exponentially weighted moving average (EWMA) was the most potential control chart to monitor both AR (1) and IMA (1, 1) processes since it is sensitive to small and large shift sizes. It is important to note that practitioners should fully understand how SPC responds to autocorrelated disturbances with deterministic shifts in order to achieve the highest performance.

Keywords: Autoregressive Integrated Moving Average, Exponentially weighted moving average, Moving range, Non-stationary, Stationary.

1. INTRODUCTION

Statistical process control (SPC) is a procedure which focuses on process monitoring and control by separating common causes from assignable causes. Common causes are the source of variation that is inherent in the process and cannot be eliminated when the process is in statistical control, while assignable cause variation is unpredictable but can be easily detected and removed. The traditional tools of SPC are Shewhart control charts, which are based on the assumptions that all processes are in-control and observations are independent. However, because of the advanced measurement technology and shortened sampling interval, the independence of each observation has been violated in many scenarios, especially in continuous process industries, e.g., chemical process. The lack of independence among each sample always comes in the form of serial correlation, which can be either positively or negatively correlated. This behavior of process outputs will significantly downgrade the performance of control charts. As a result, the control limits of control charts will be narrower than what they should be and might signal false alarms more frequently. The consequence is the unnecessary investigation, which consumes a lot of time and money. Therefore, several authors point out that the traditional charts fail to control and improve the quality of correlated processes [Zhang, 1998, Jiang, Tsui and Woodall, 2000 and Loredo, Jearkpaporn and Borror, 2002].

2. LITERATURE REVIEW

Deming (1998) stated that the effort to adjust a stable (in-control) process in order to compensate for an undesirable disturbance tampered the process and led to more variation, so it was better to leave the process alone. Nonetheless, if the process was left uncontrolled, the process output was stationary with highly correlated data, or non-stationary due to disturbances. This might cause the process mean to wander from the desired target.

One of the solutions to this problem is the integration of forecasting models with the traditional SPC tools, since they have the capability to describe the correlation structure of the data [Loredo, et al., 2002]. Most authors proposed that the integration between forecasting techniques and SPC can be done by predicting the process mean in the future, so the modified charts (model-based control charts) can adapt their control limits to the correlation by following the forecasts.

Wardell, Moskowitz and Plante (1992) suggested that the integration of forecasting methods and the SPC was done in the form of model-based control charts because of the ability to keep track of the correlation pattern. The model-based control charts were constructed on the idea that almost all process data was fitted by the autoregressive moving average (ARIMA) model but, instead of using it to filter the data, the normal control limits of Shewhart charts were replaced by the variance of the disturbances (based on ARIMA model).

Another approach (data filtering technique) was based on the idea that the errors from forecasting models were monitored by the traditional charts to detect the assignable causes after the residual satisfied all the assumptions. Alwan and Roberts (1988) and Montgomery and Mastrangelo (1991) recommended that one of the forecasting applications which supported the utilization of control charts was data filtering, since the correlation embedded in the observations was removed (filtered) by fitting the appropriate forecasting model to the correlated data. After the proper model was applied to the data, the residual from the filtering was highly likely to be identically, independently and normally distributed (i.i.d.). As a result, SPC control charts were then used to monitor the residual and to detect any outliers, since all the conditions were satisfied. If a shift occurred in the process, there was a shift in the mean of the residuals. For example, Lu and Reynolds (1999) utilized an ARIMA model and an exponentially weighted moving average (EWMA) chart to monitor the residuals, based on the ARIMA model forecast values.

Autoregressive integrated moving average (ARIMA) model was a stochastic difference equation that was frequently utilized to model stochastic disturbances. The general form of ARIMA model was shown in equation (1).

$$\Delta_d Y_t = \mu + \phi_1 \Delta_d Y_{t-1} + \phi_2 \Delta_d Y_{t-2} + \ldots + \phi_p \Delta_d Y_{t-p} + a_t - \theta_1 a_{t-1} - \ldots - \theta_q a_{t-q} \tag{1}$$

The order of ARIMA model was normally identified in the form of (p, d, q). p indicated the order of the autoregressive part while d was for the amount of difference and q for the order of the moving average part. Some specific form of ARIMA model was utilized to represent autocorrelated disturbances, e.g. autoregressive order one, ARIMA (1, 0, 0) or AR (1) for stationary disturbances while integrated moving average, ARIMA (0, 1, 1) or IMA (1, 1) was used to represent non-stationary disturbances, as recommended by Montgomery, D.C., Keats, J.B., Runger, G.C. and Messina (1994) and Box and Luceno (1997).

Since there were two implementation techniques (data filtering and model-based control chart) which were accepted among authors, several experimental studies were conducted in order to access the performance of each approach [Wardell, et al., 1992]. In a study, a set of correlated data was numerically simulated by following the ARIMA (1, 1) model, while the step input signal was applied to the data periodically in order to represent the special cause (or assignable cause) in the process. The performance measurement of this experiment is based on the average run length (ARL) when there was a shift (special cause) in the system. Due to the experimental result, the model based control charts were sensitive to the small shift sizes when the autoregressive (AR) term was negative and the moving average (MA) term was positive. However, when the shift size was large, the implementation of the data filtering technique showed that it signaled an out-of-control status much faster than the model-based one. According to the comparison, none of these methods were superior over another in every situation, but most authors suggested that the model-based control chart might be preferred since it was simple to implement.

Another critical problem is that most observations had no fixed patterns over a period of time so the success of the monitoring also depended on the accuracy of the forecasting. In addition, some practitioners might find that the above two techniques were too complicated and difficult to implement in real-life situations. For this reason, Jiang et al. (2000) proposed the third approach which was the utilization of the classical Shewhart chart by selecting the most appropriate control charts to monitor correlated observations directly under different conditions. This initiative was supported by the work of English, Lee, Martin and Tilmon (2000) which compared the performance of $\overline{X}$ and EWMA charts when the processes were autoregressive.

However, the contribution regarding the use of SPC alone was still limited so the identification of suitable control charts for monitoring autocorrelated processes following different types of ARIMA models was a widely discussed issue in the literature. As a result, if the performance of the each chart under the autocorrelated scenarios is known, practitioners will have guidelines for achieving the highest capability when deploying SPC and ARIMA model.

3. SIMULATION MODELS

In this research, a simulation model comprised the disturbance generator and control charts were utilized to quantify the effect of autocorrelation on SPC. The autocorrelation was categorized into two cases: stationary and non-stationary disturbances. The stationary disturbances were represented in the form of autoregressive order one model, AR (1), while the non-stationary ones were characterized by integrated moving average model, IMA (1, 1). The level of autocorrelation was adjusted by controlling AR coefficient (ϕ) and MA coefficient (θ) while a step function generator was utilized to signify the deterministic disturbance. The diagram which depicted the simulation model was shown in Figure 1.

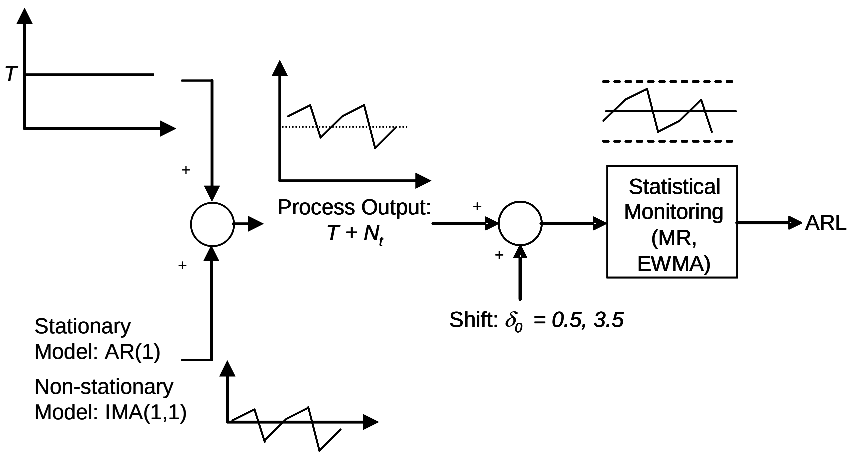

Figure 1. Simulation model

The moving range (MR) and exponentially weighted moving average (EWMA) charts were utilized to monitor the autocorrelated observations Y. The performance of each chart was assessed by measuring the average run length (ARL) which indicated the average number of points that was plotted on a control chart before an out of control condition was acknowledged. The process observation (Y_{t+1}) was represented by

$$Y_{t+1} = T + N_{t+1} + \delta(t)\,. \tag{2}$$

The source of stationary autocorrelation was characterized by the autoregressive order one, AR (1), as follows:

$$N_{t+1} = \phi N_t + a_{t+1};\ -1 < \phi < 1, \tag{3}$$

On the other hand, the integrated moving average model, IMA (1, 1) was utilized to represent non-stationary disturbances as follows:

$$N_{t+1} - N_t = a_{t+1} - \theta a_t;\ -1 < \theta < 1, \tag{4}$$

where N_{t+1}, N_t are disturbances at time t+1 and t respectively, a_{t+1}, a_t are random errors at time t+1 and t respectively, ϕ is the autoregressive (AR) parameter and θ is the moving average (MA) parameter.

When a special cause occurred at time t_0, a step function of magnitude size δ_0 was utilized to represent a shift as shown in equation (5).

$$\delta(t) = \begin{cases} 0; t < t_0 \\ \delta_0; t \geq t_0 \end{cases}, \tag{5}$$

where δ_0 is the magnitude of a shift, t_0 is the time that a shift occurs.

The observations were monitored by a Shewhart moving range (MR) chart and an exponentially weighted moving average (EWMA) chart. The control limits for a moving range chart were

$$\left.\begin{array}{c} Upper\ Control\ Limit\ (UCL) = \overline{Y} + 3\dfrac{\overline{MR}}{d_2} \\ Center\ line\ (CL) = \overline{Y} \\ Lower\ Control\ Limit\ (LCL) = \overline{Y} - 3\dfrac{\overline{MR}}{d_2} \end{array}\right\}, \tag{6}$$

where $\overline{Y}$ is the process mean and equals $(\sum_{t=1}^{n} Y_t)/n$, $MR = |Y_t - Y_{t-1}|$, $\overline{MR}$ is the average of moving average, d_2 = 1.128 (the moving range of n = 2 observations).

For an *EWMA* chart, the control limits were expressed as:

$$\left.\begin{array}{c} UCL = \mu_0 + L\sigma\sqrt{\dfrac{\lambda}{(2-\lambda)}[1-(1-\lambda)^{2i}]} \\ CL = \mu_0 \\ LCL = \mu_0 - L\sigma\sqrt{\dfrac{\lambda}{(2-\lambda)}[1-(1-\lambda)^{2i}]} \end{array}\right\}, \tag{7}$$

where μ_0 is the average of preliminary data, L is the width of control limits, λ is the weight assigned to the observation. The values of L and λ used were recommended by Lucas and Saccucci (1990).

4. EXPERIMENTAL PROCEDURES

An empirical analysis was conducted using an experimental design package, Design Expert® Version 8.0, to analyze the effect of the input factors on the responses. The analysis of variance (ANOVA) and the half-normal plot were utilized to reveal the significant factors and their interactions. The analysis was categorized into two cases based on the stationarity of the observations. For stationary case, the selected factors to be investigated were AR parameters (ϕ), shift sizes and types of control charts. The response was average run length (ARL). Since the factorial design 2^k was used to empirically analyze the relationship between input factors and response, ϕ were set to -1 and 1 for low and high values respectively. Additionally, shift sizes of 0.5 σ_a and 3.5 σ_a were deployed to represent small and large shifts. Two types of control charts, a Shewhart moving range (MR) chart and an exponentially weighted moving average (EWMA) chart, were used in order to monitor processes. In conclusion, each factor for stationary case was set to high and low levels as shown in Table 1.

Table 1. Input Factors and Levels (Stationary Case).

Factor	Low	High
A (AR parameter; ϕ)	-1	1
B (Types of charts)	MR	EWMA
C (Shift size)	$0.5\sigma_a$	$3.5\sigma_a$

Similarly, the input factors and their levels for non-stationary processes were shown in Table 2.

Table 2. Input Factors and Levels (Non-stationary Case).

Factor	Low	High
A (MA parameter; θ)	-1	1
B (Types of charts)	MR	EWMA
C (Shift size)	$0.5\sigma_a$	$3.5\sigma_a$

Regarding the simulation, each run was composed of 10,000 iterations which have been accomplished using Palisade's @Risk® Version 5.5. The random errors (a_t) from each period were simulated by following normal distribution with zero mean and a constant variance as: $a_t \sim N(0, \sigma_a^2)$. The simulation results and the analysis of ARL response for each case were shown in the following section.

5. EXPERIMENTAL RESULTS

The results were categorized into two cases as follows:

5.1 Stationary case; AR (1)

The design matrix and the experimental results for the stationary case were shown in Table 3.

Table 3. ARL Response (Stationary Case).

Run	ϕ	Shift	Chart	ARL
1	-1	0.5	MR	46.4278
2	1	0.5	MR	2.7242
3	-1	3.5	MR	43.304
4	1	3.5	MR	1.3198
5	-1	0.5	EWMA	1.6502
6	1	0.5	EWMA	1.8863
7	-1	3.5	EWMA	1.1673
8	1	3.5	EWMA	1.2164

The half-normal plot (Figure 2) and analysis of variance (ANOVA) in Table 4 showed that type of chart (C), AR model coefficients (A) and their interaction (AC) contributed the significant effects on the ARL. The experimental results also indicated that EWMA chart was robust to both outliers and the correlation structure of the observations because of its low ARLs. The interaction plot AC in Figure. 3 showed that there was no difference for the ARL when EWMA chart was deployed to monitor processes. On the other hand, MR chart was sensitive to the stationary autocorrelation when ϕ was highly negative because the ARL at ϕ = -1 was significantly higher than the one at ϕ = +1. This result explicitly identified that MR chart was a suitable chart to monitor autocorrelated observations with highly positive AR parameter.

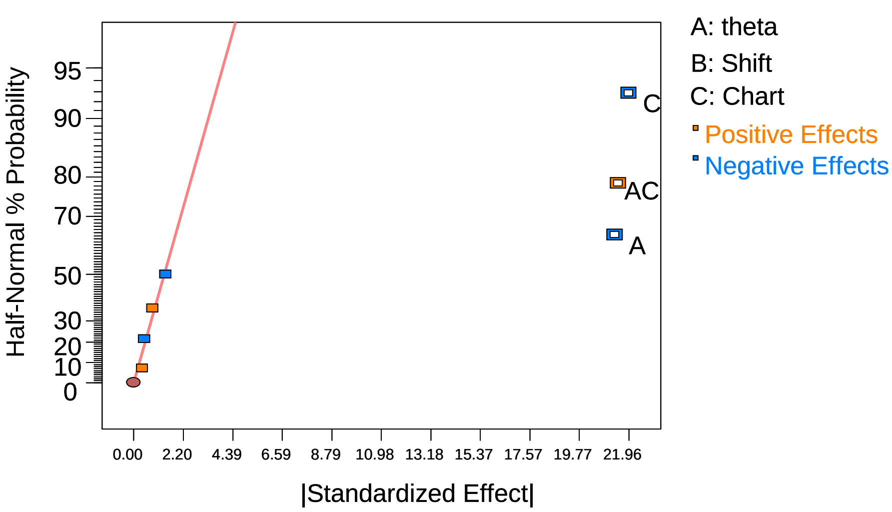

Figure 2. Half-normal Plot (Stationary Case).

Table 4. ANOVA (Stationary Case).

Source	SS	Df	MS	F	p-value
Model	2800.446	3	933.482	601.6436	< 0.0001
A-ϕ	911.7005	1	911.7005	587.6051	< 0.0001
C-Chart	964.8258	1	964.8258	621.8452	< 0.0001
AC	923.9196	1	923.9196	595.4805	< 0.0001
Residual	6.206212	4	1.551553		
Total	2806.652	7			

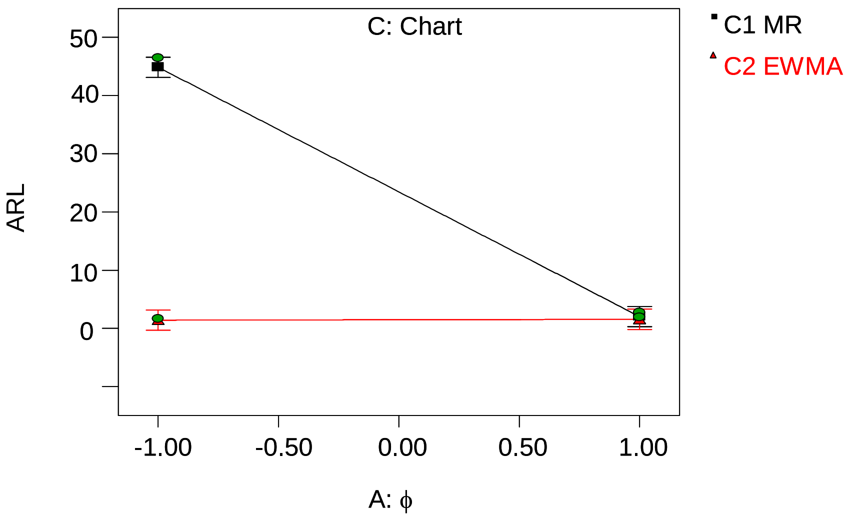

Figure 3. Interaction Plot (AC).

5.2 Non-stationary case; IMA (1, 1)

The design matrix for the non-stationary case and the results were shown in Table 5. For non-stationary case, after the regression equation was constructed, the transformation was required to ensure that residuals satisfied the i.i.d conditions. After applying the natural logarithm transformation to the response ARL, the ANOVA and the half-normal plot were utilized to reveal the significant factors and their interactions as shown in Figure 4 and Table 6 respectively.

Table 5. ARL Response (Non-stationary Case).

Run	θ	Shift	Chart	ARL
1	-1	0.5	MR	2.3325
2	1	0.5	MR	5.4765
3	-1	3.5	MR	1.3765
4	1	3.5	MR	1.3185
5	-1	0.5	EWMA	1.3589
6	1	0.5	EWMA	2.8043
7	-1	3.5	EWMA	1.1572
8	1	3.5	EWMA	1.0252

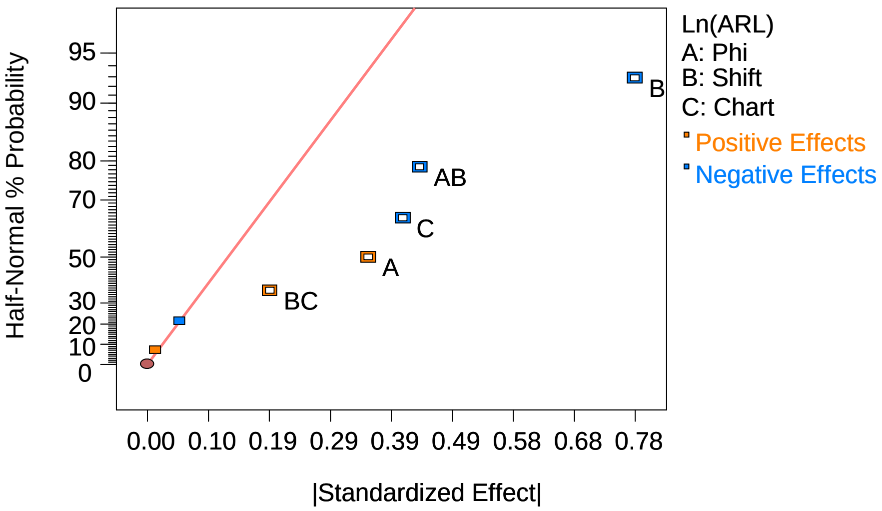

Figure 4. Half-normal Plot (Non-stationary Case).

Table 6. ANOVA (Non-stationary Case).

Source	SS	Df	MS	F	p-value
Model	2.255698	5	0.45114	158.6599	0.0063
A-Phi	0.249868	1	0.249868	87.87516	0.0112
B-Shift	1.215479	1	1.215479	427.4682	0.0023
C-Chart	0.334041	1	0.334041	117.4778	0.0084
AB	0.379394	1	0.379394	133.4279	0.0074
BC	0.076916	1	0.076916	27.05043	0.0350
Residual	0.005687	2	0.002843		
Total	2.261385	7			

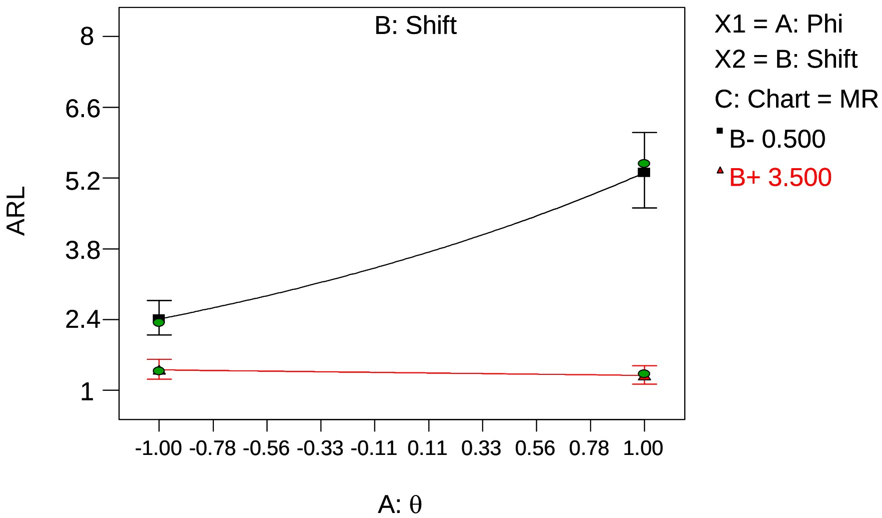

Figure 5. Interaction Plot (AB) for MR chart.

According to Figure 5, 6, 7 and 8, they revealed that both EWMA and MR chart was sensitive to small shift size only when MA parameter (θ) was highly negative. Similar to the results from stationary case, EWMA chart should be selected to monitor processes since its ARLs were lower than those of MR chart in every scenarios.

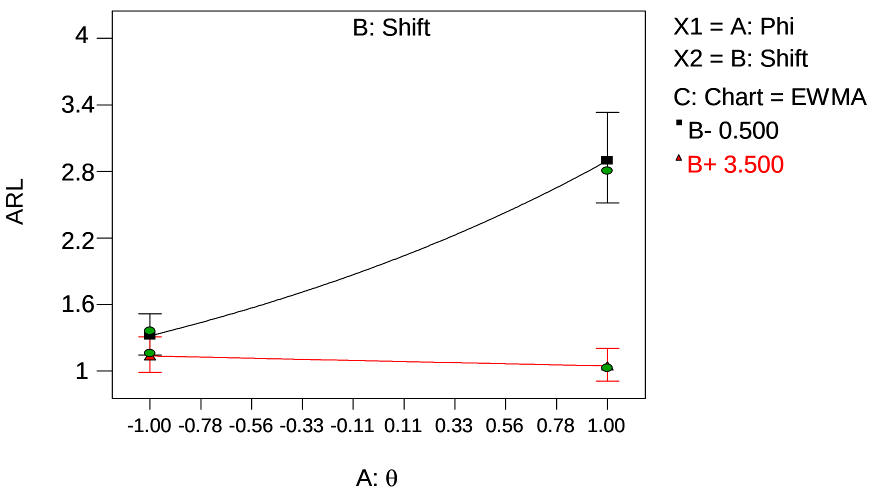

Figure 6. Interaction Plot (AB) for EWMA Chart.

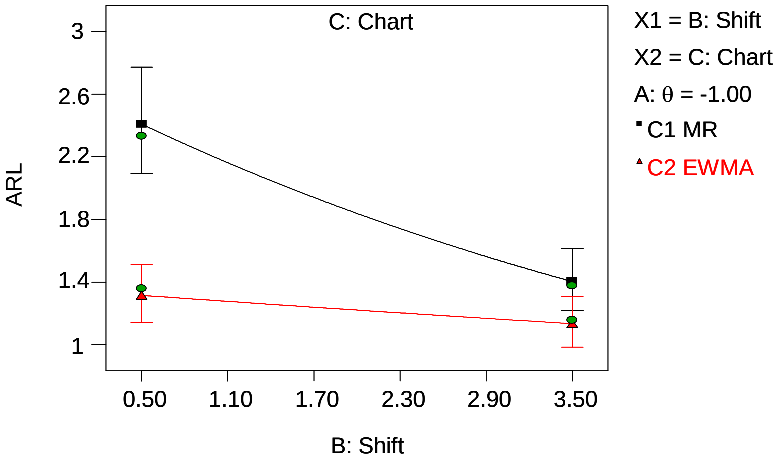

Figure 7. Interaction Plot (BC) for θ = -1.00.

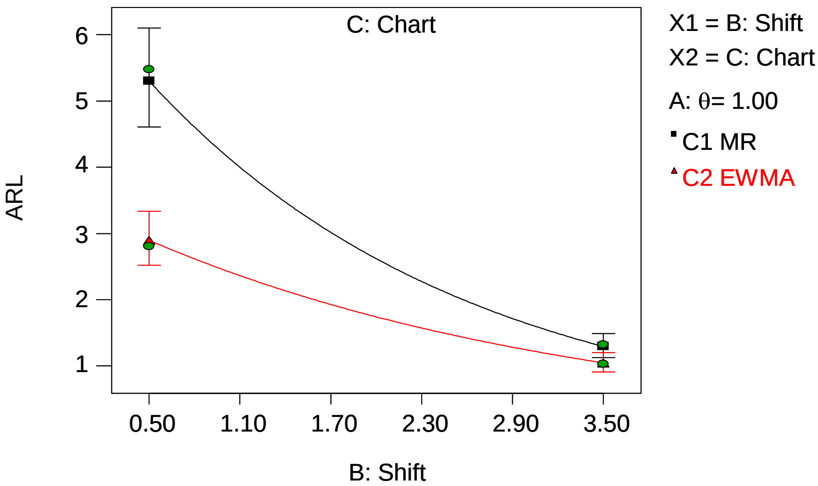

Figure 8. Interaction Plot (BC) for θ = +1.00.

6. CONCLUSIONS

This research focused on the performance analysis of a statistical process control system in order to quantify the effects of the selected factors on stationary and non-stationary processes. According to the analysis, the effects of AR parameter (ϕ), MA parameters (θ), appropriate types of control charts and shift sizes on the ARL were determined. In summary, the resultant analysis was concluded as follows:

1. When the observations follows AR (1) pattern, shift size does not have any significant effects on the ARL. Anyway, the AR coefficient seems to play an important role on the selection of SPC charts. The empirical analysis reveals that EWMA is the most suitable control chart to monitor stationary processes because of its robustness. However, MR chart can also be utilized in the scenario that ϕ is highly negative.

2. When IMA (1, 1) was utilized to characterize the non-stationary processes and θ was highly negative, both EWMA and MR chart was sensitive to small shift size.

3. For both stationary and non-stationary cases, the performance of the SPC to minimize ARL will be significantly improved if an EWMA chart is utilized to monitor the observations.

According to the results, the selection of appropriate control charts will assist practitioners to monitor the autocorrelated processes effectively.

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